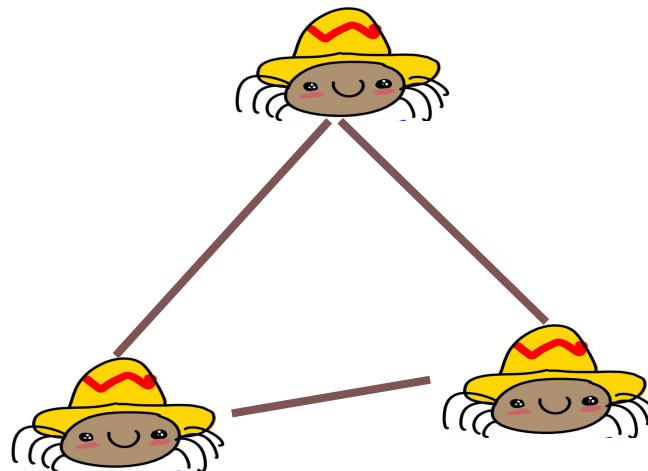




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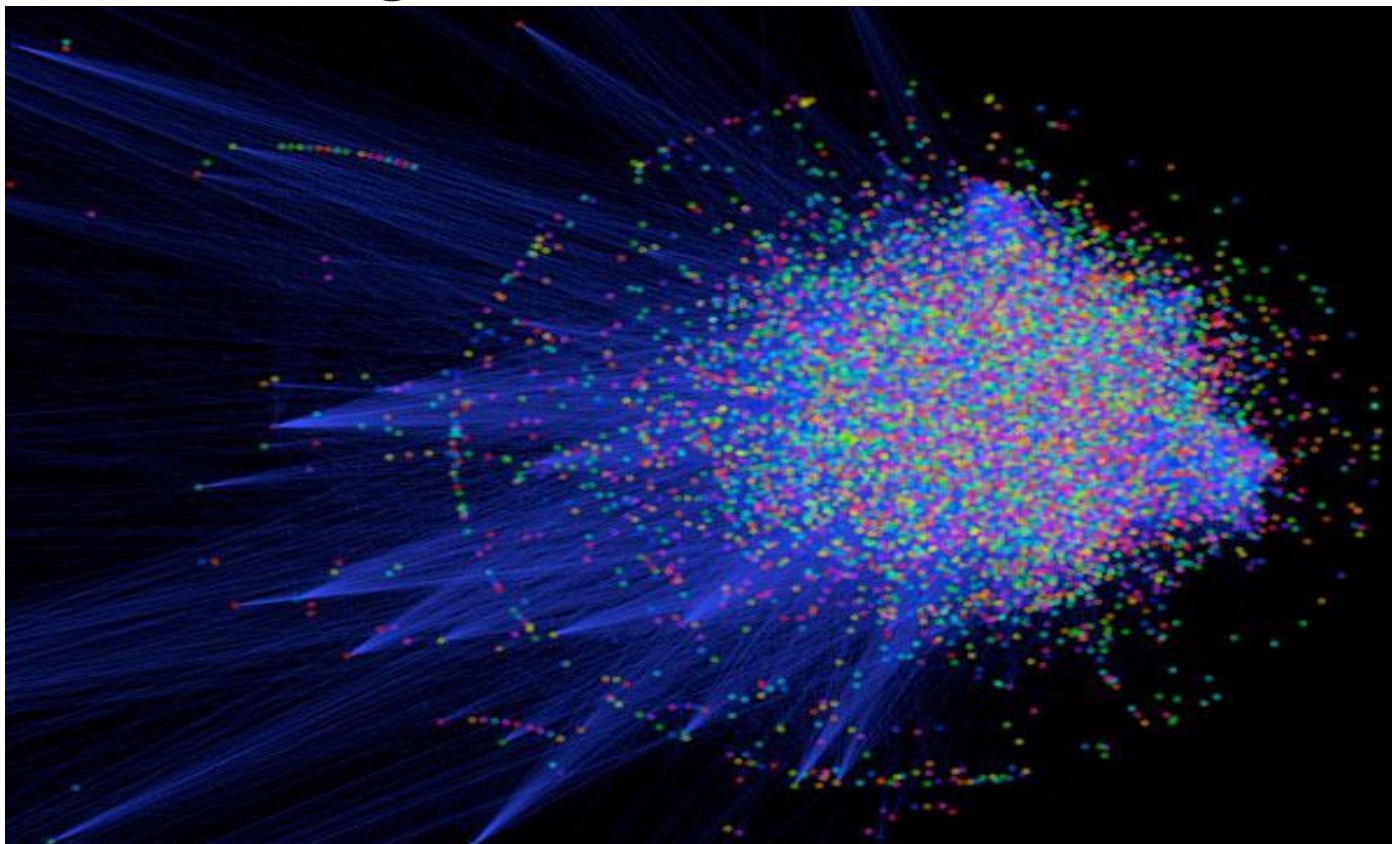


# MODELO GENÉRICO DE REDE

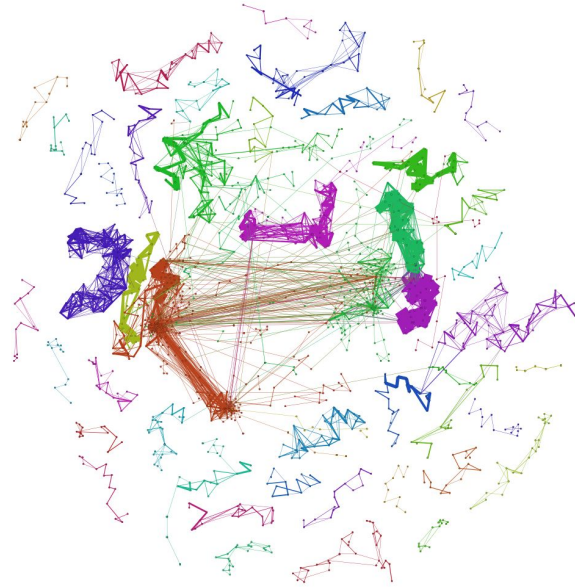
---

Prof. Fabrício Olivetti de França

# Modelos genéricos de rede



# Modelos genéricos de rede



# Por que criar um modelo genérico de rede?

- ❑ Próximas conexões de transporte público?
- ❑ Quais conexões serão criadas ou desfeitas em relações comerciais?
- ❑ Qual a dinâmica de uma rede social?



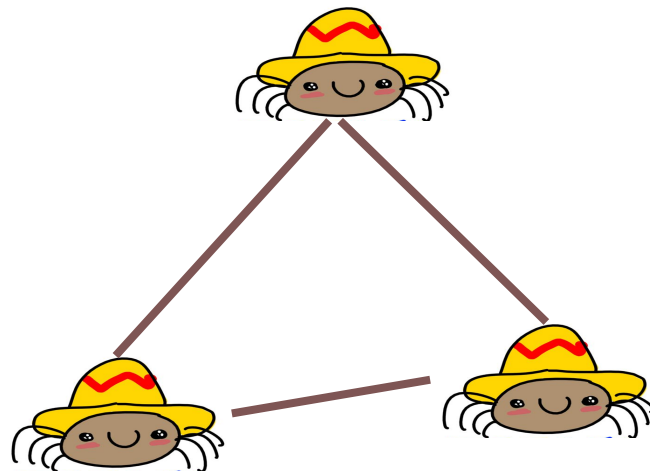
# Por que criar um modelo genérico de rede?

Precisamos conhecer as propriedades de redes de interesse e modelar matematicamente para melhor compreensão.





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# REDES ALEATÓRIAS: ERDŐS-RÉNYI

---

Prof. Fabrício Olivetti de França

# Redes Aleatórias



Paul Erdős



Alfred Rényi



# Redes Aleatórias

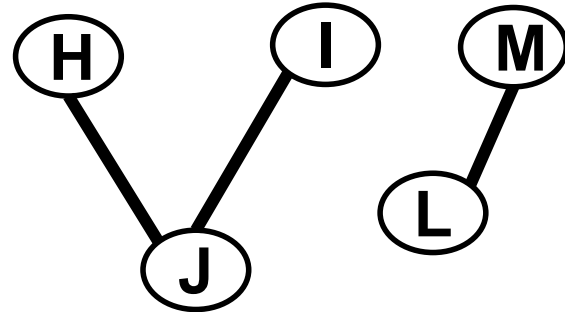
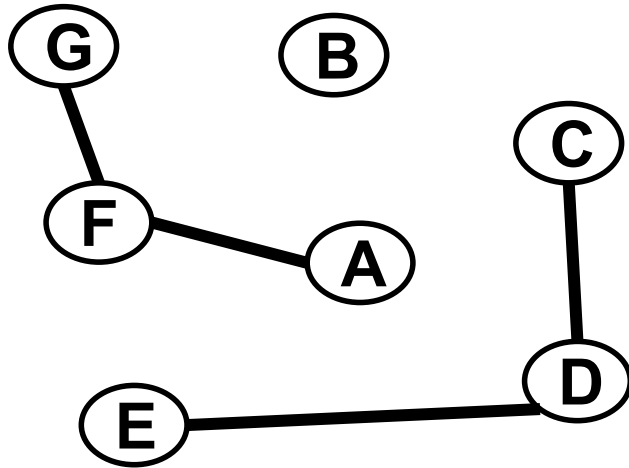




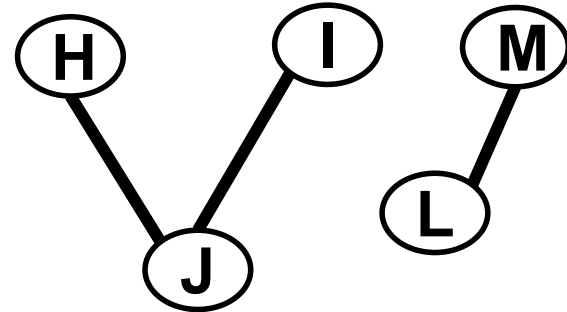
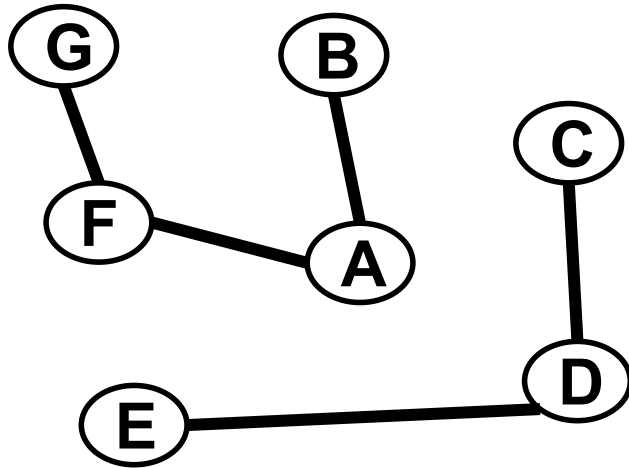
# Redes Aleatórias



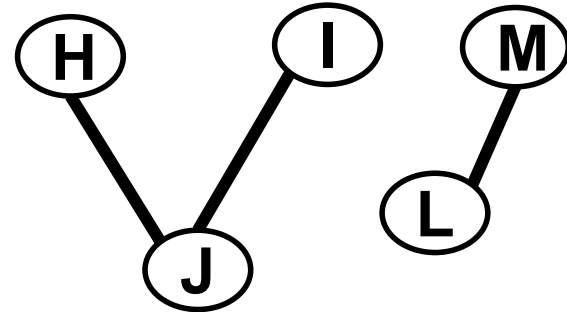
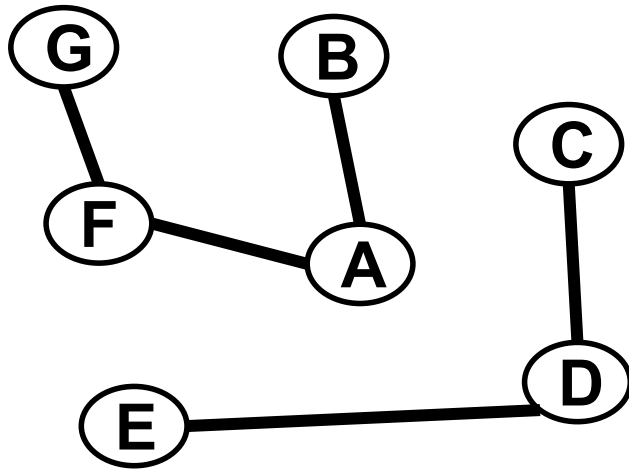
# Redes Aleatórias



# Redes Aleatórias



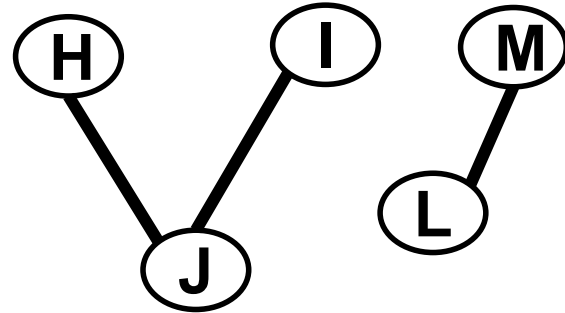
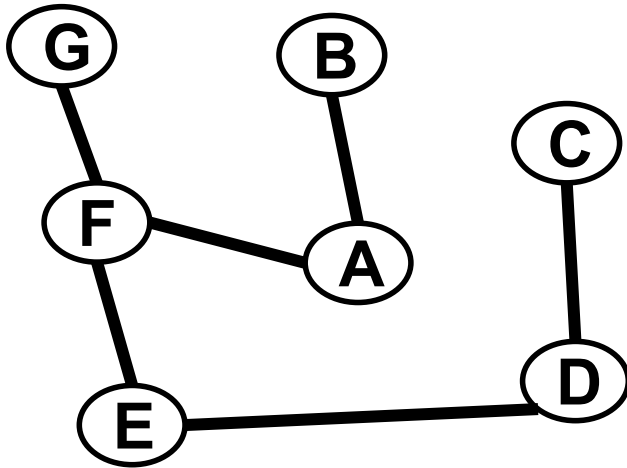
# Redes Aleatórias



Média do GRAU = 1,3  
# COMPONENTES = 4



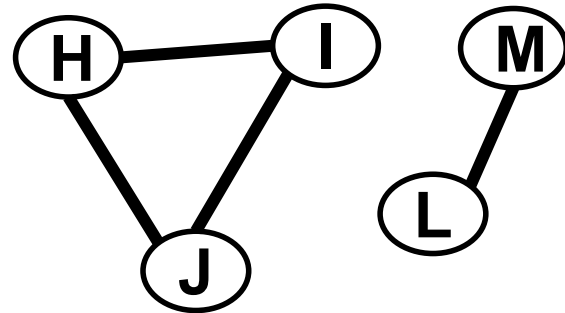
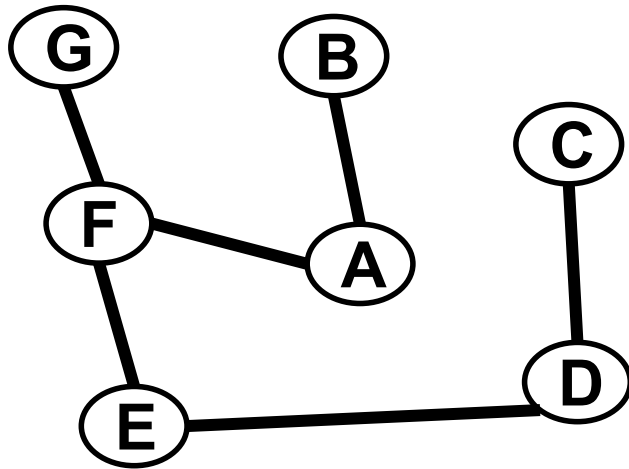
# Redes Aleatórias



Média do GRAU = 1,5  
# COMPONENTES = 3



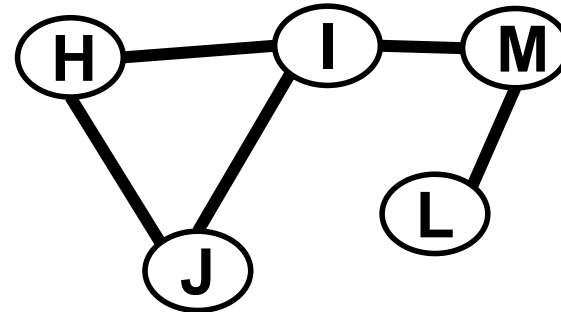
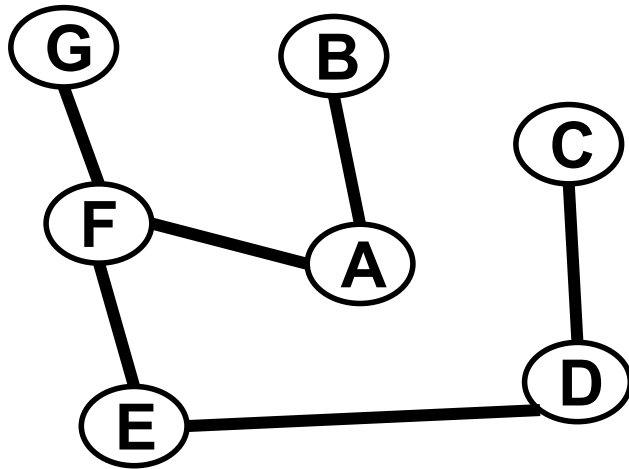
# Redes Aleatórias



Média do GRAU = 1,7  
# COMPONENTES = 3



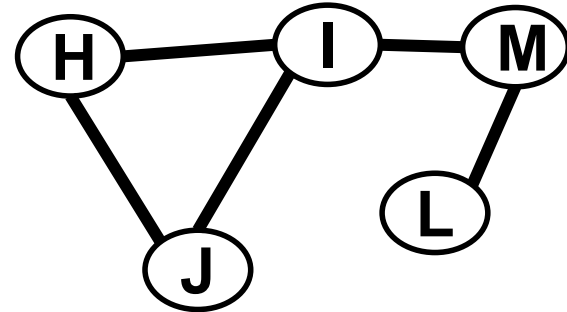
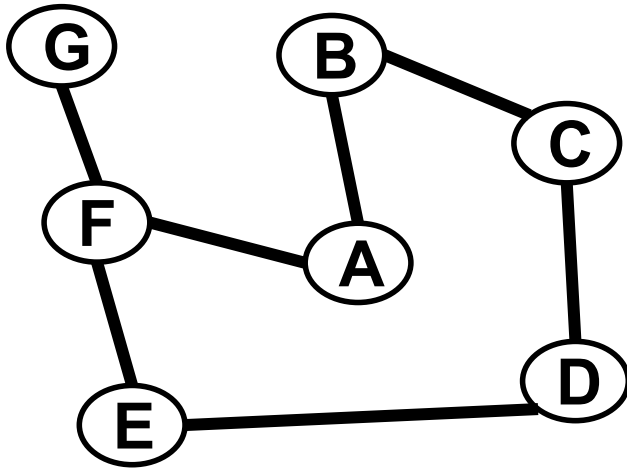
# Redes Aleatórias



Média do GRAU = 1,8  
# COMPONENTES = 2



# Redes Aleatórias

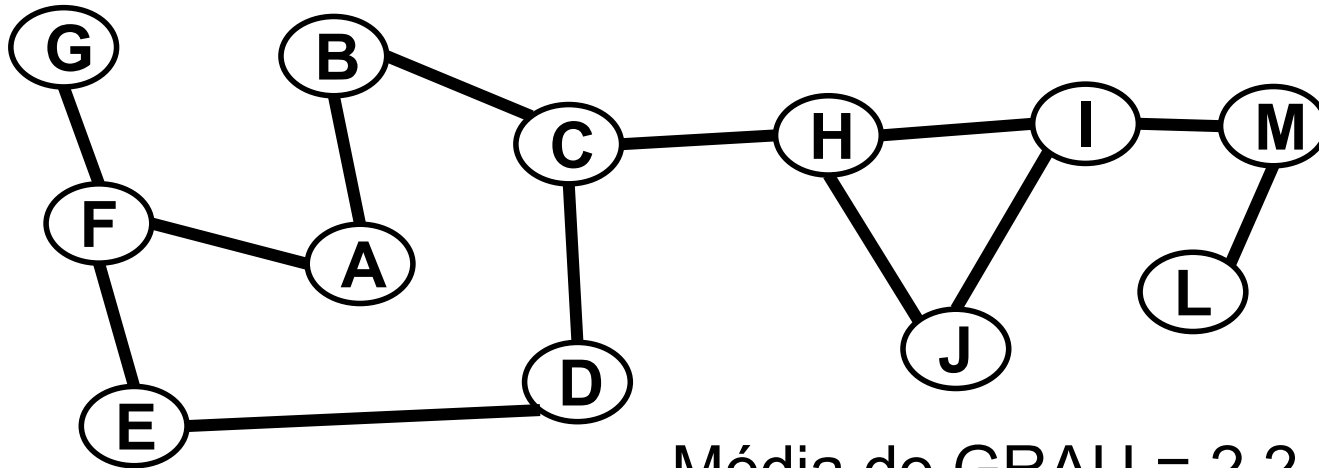


Média do GRAU = 2  
# COMPONENTES = 2



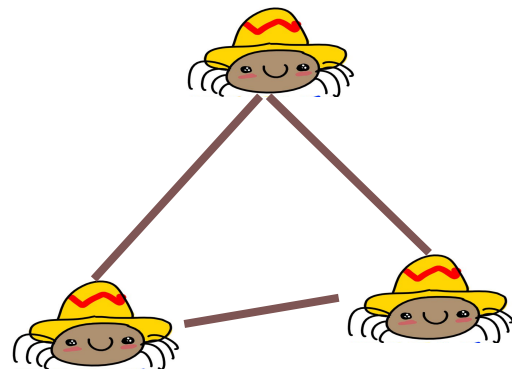
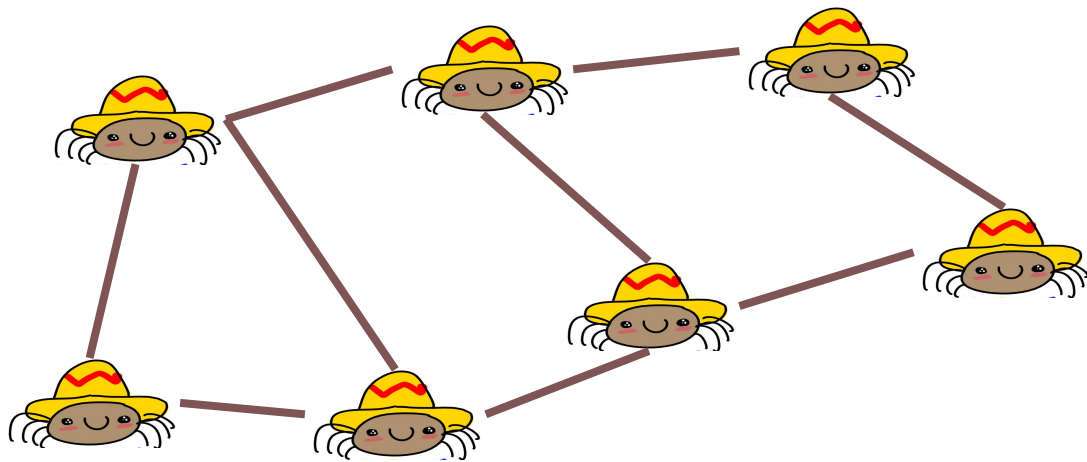


# Redes Aleatórias

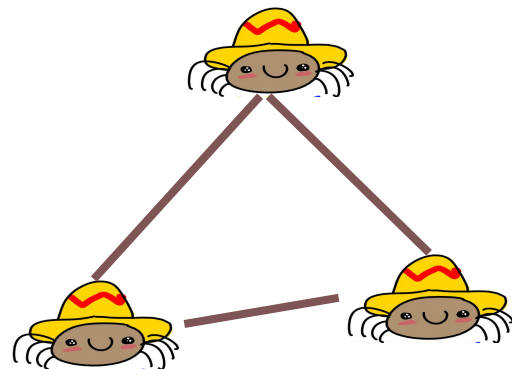
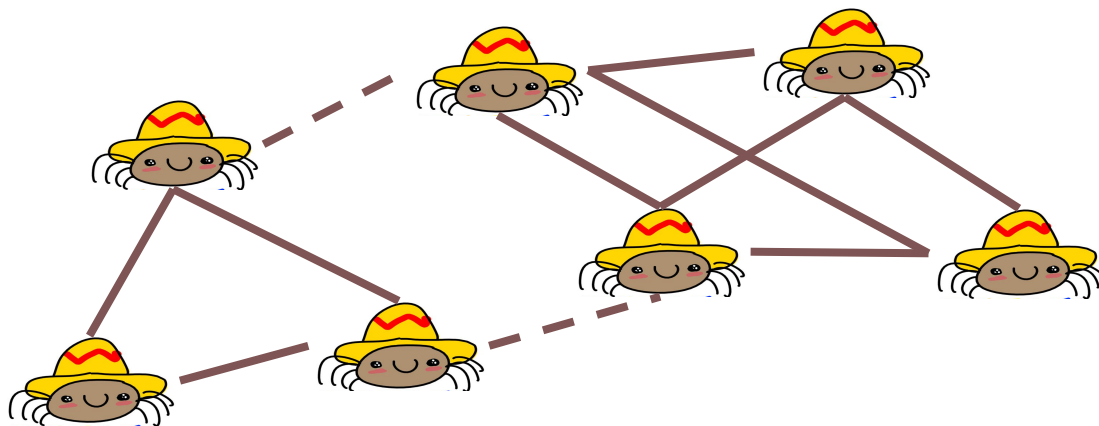


Média do GRAU = 2,2  
# COMPONENTES = 1

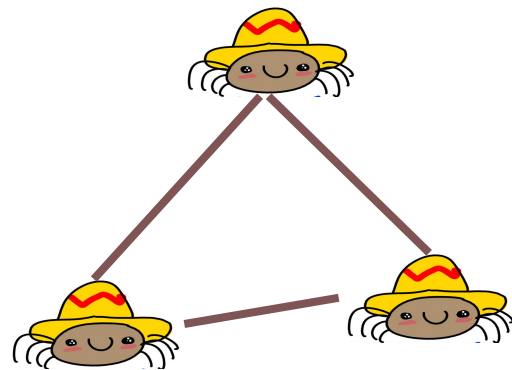
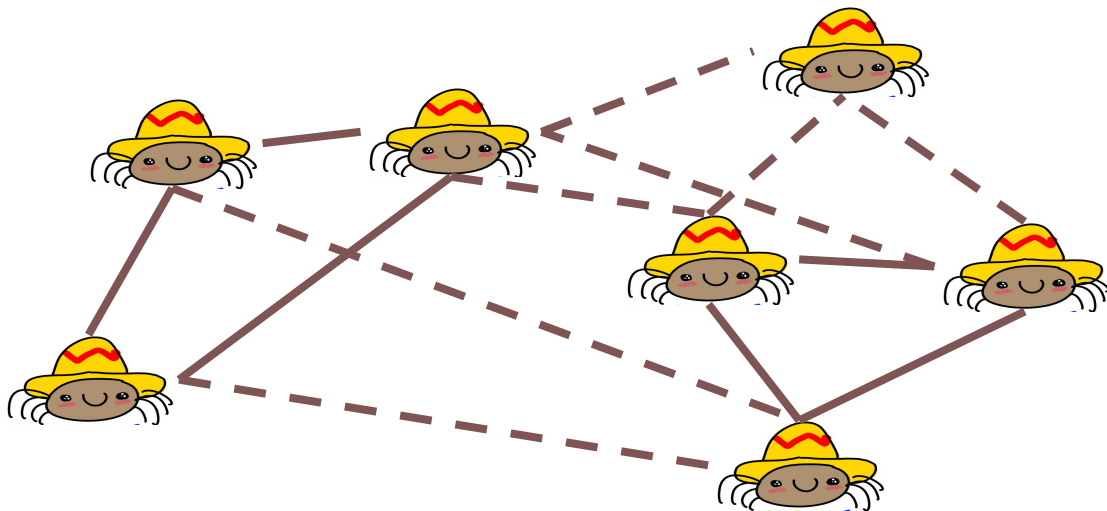




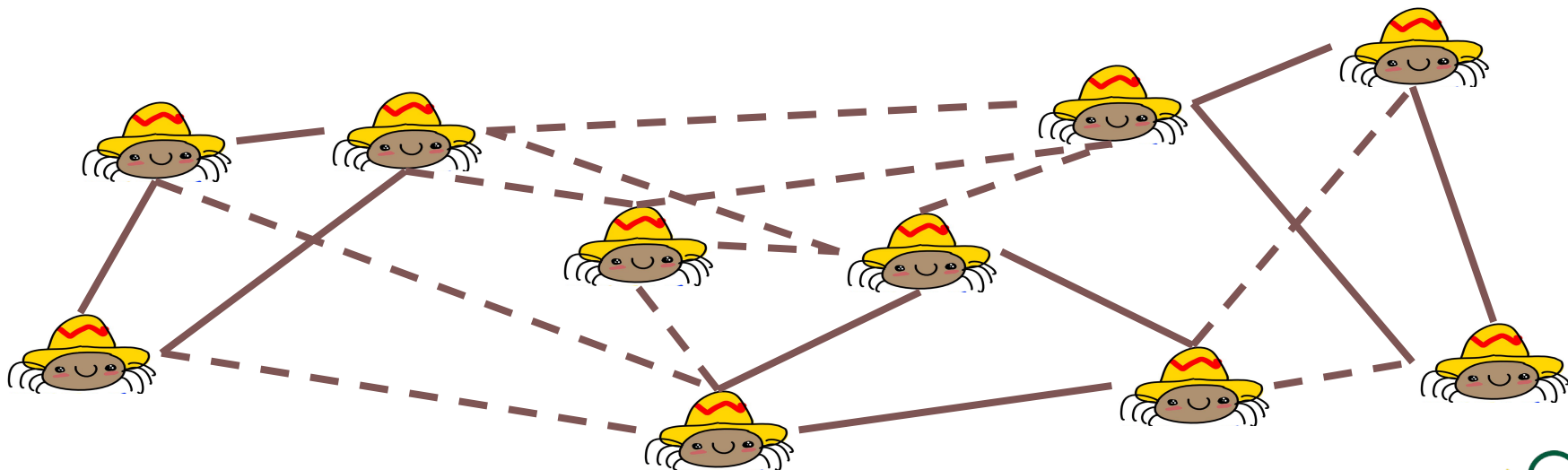
# Festa!!!!



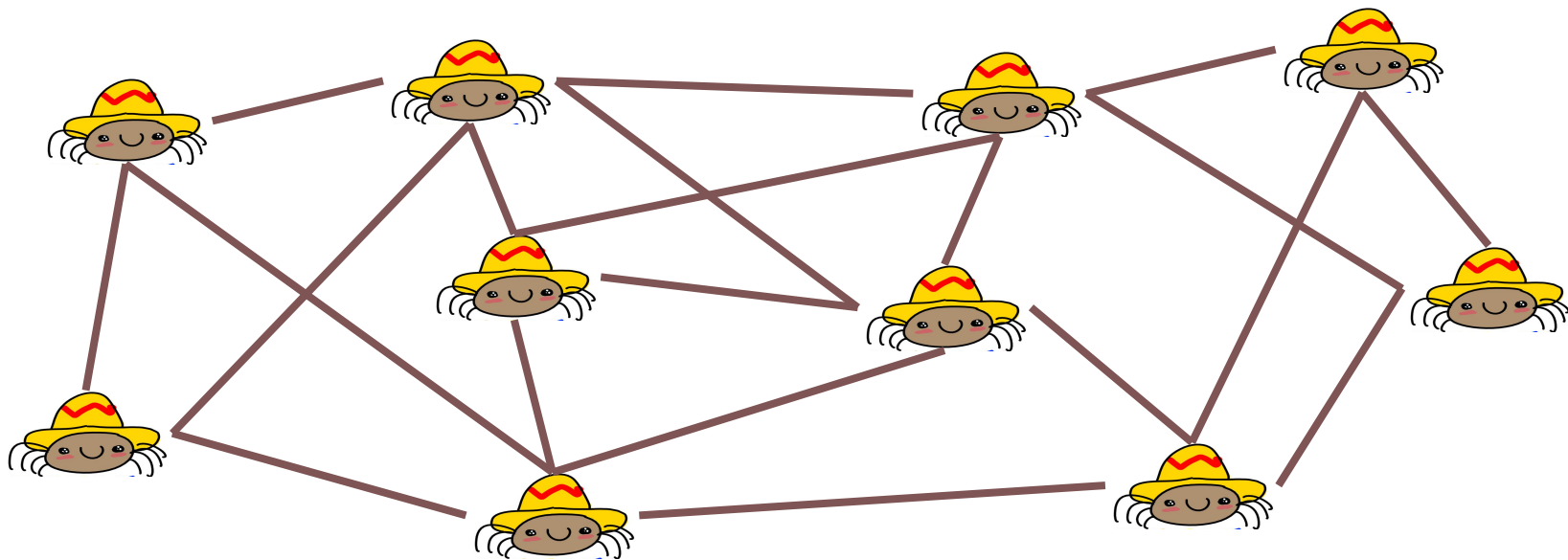
# Festa!!!!



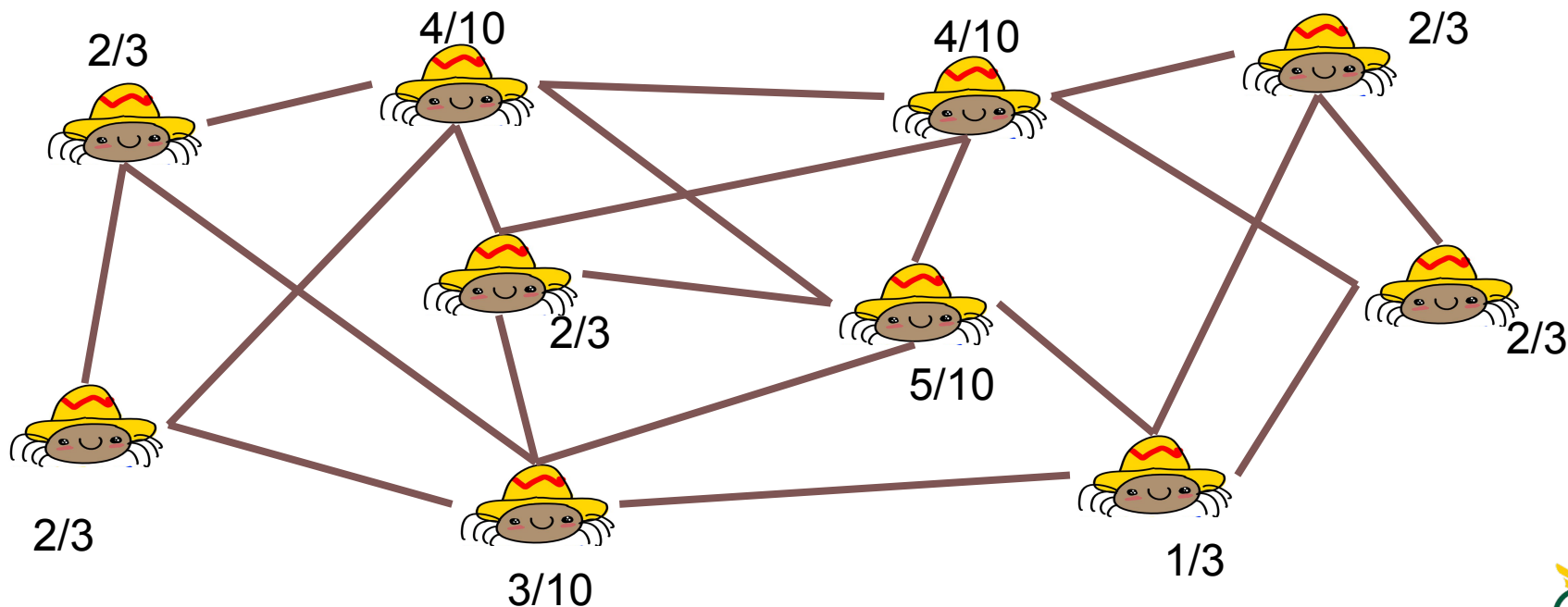
# Festa!!!!



# Festa!!!!



# Festa!!!!



Coeficiente de agrupamento médio = 0,53



# Criando uma rede aleatória

Modelo Erdős-Rényi é dado por:

$G(n,p)$

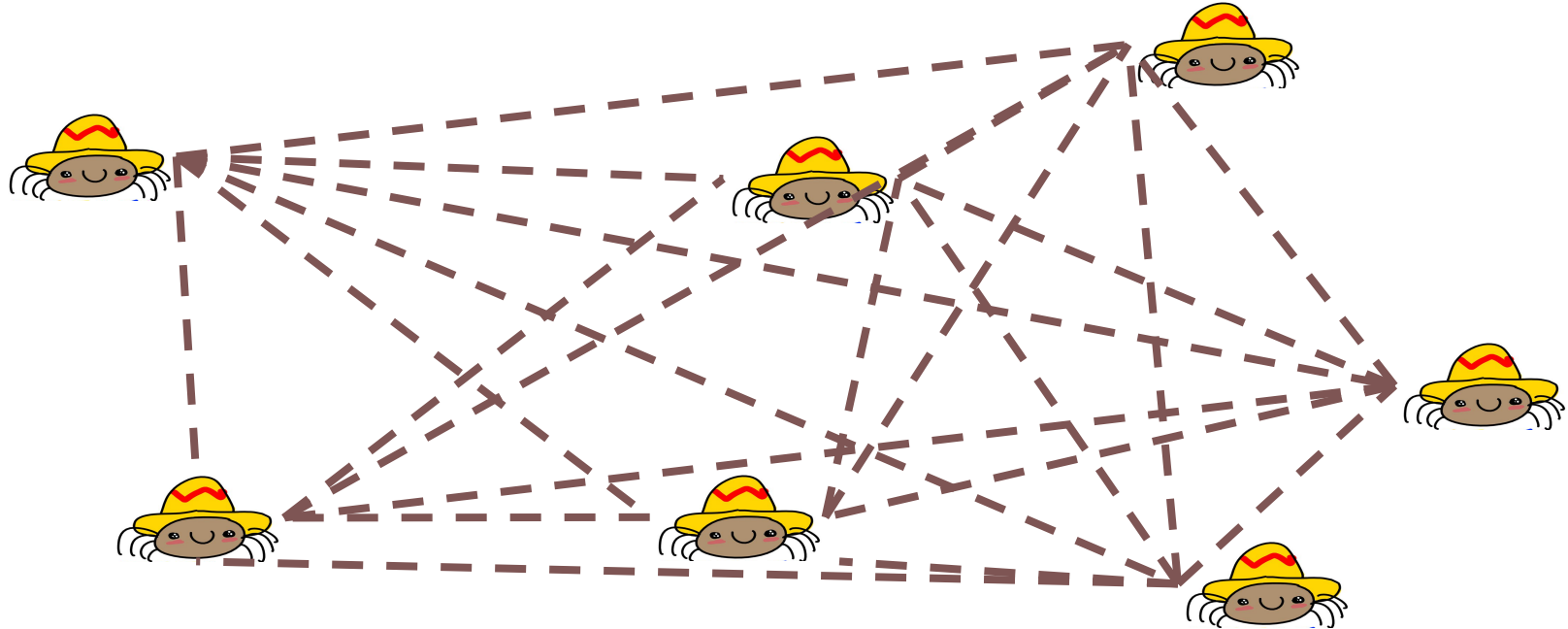
$$P = \frac{|E|}{n(n-1)}$$





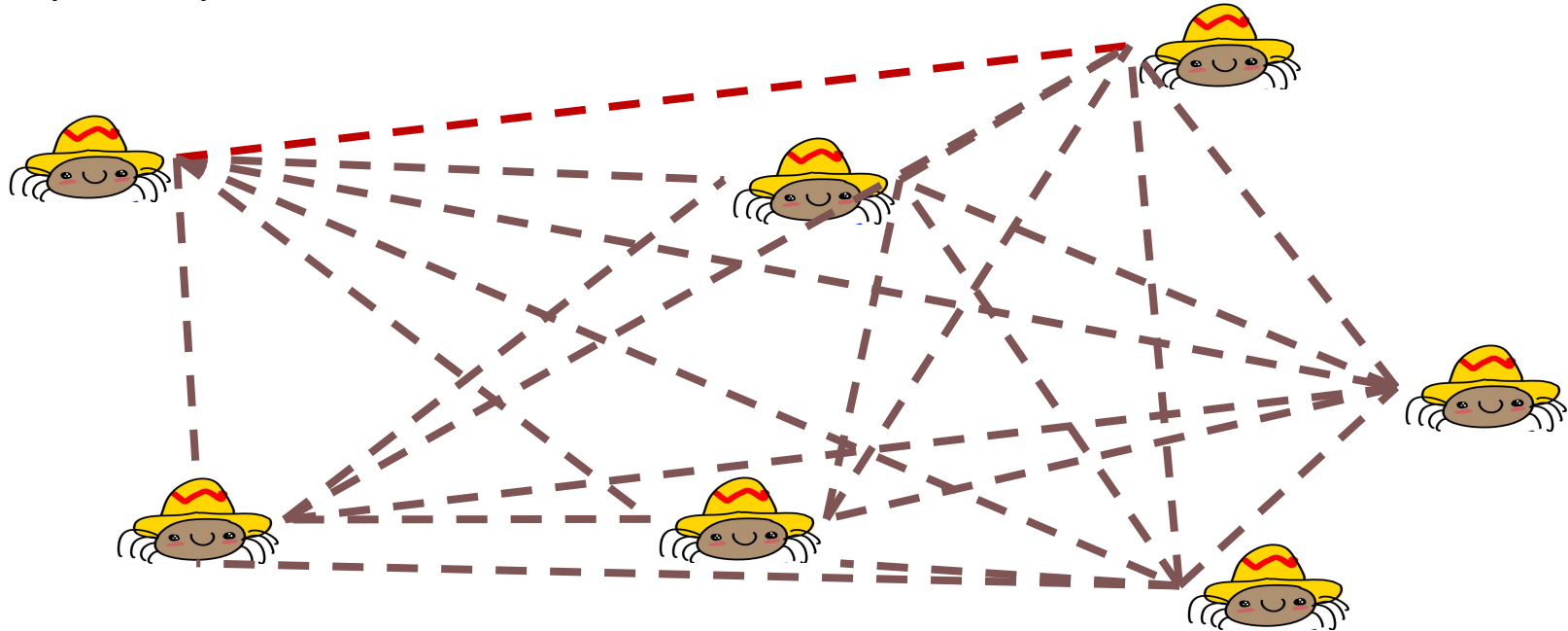
# Gerando uma rede aleatória

$G(7,0.3)$



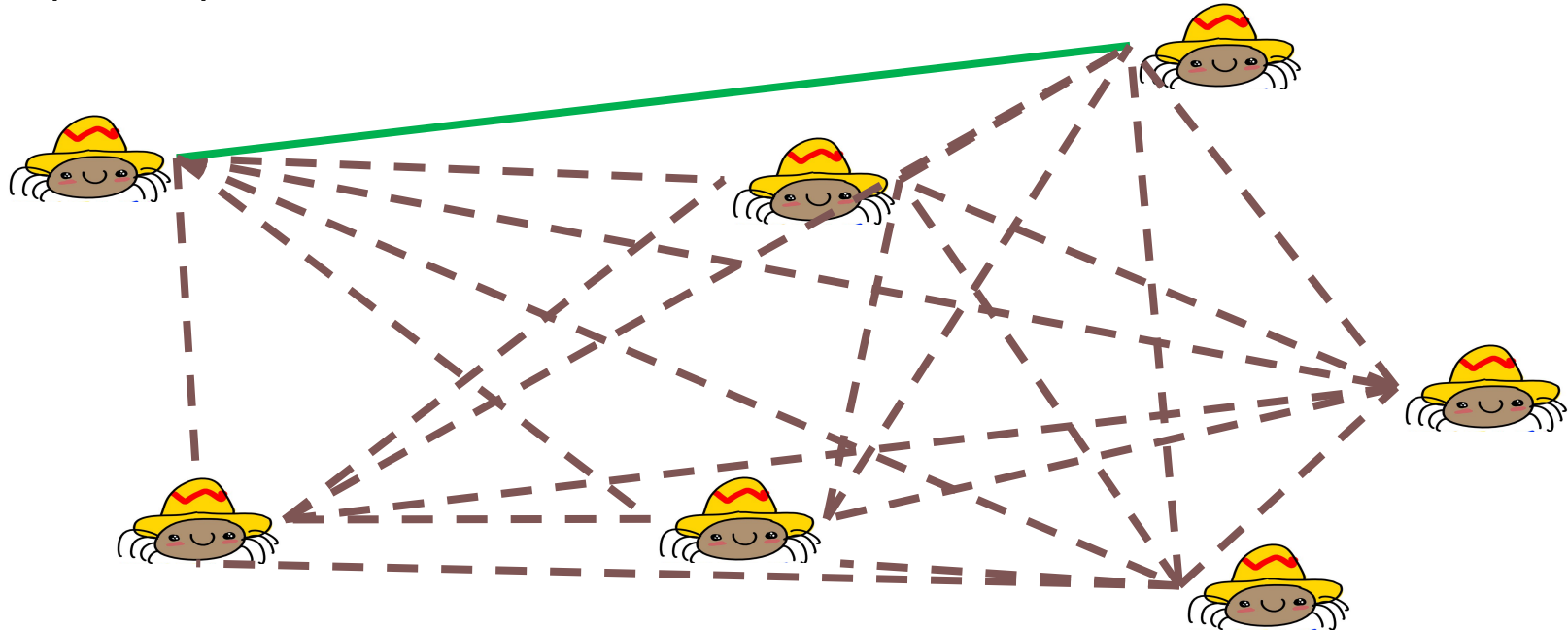
# Gerando uma rede aleatória

$G(7,0.3)$



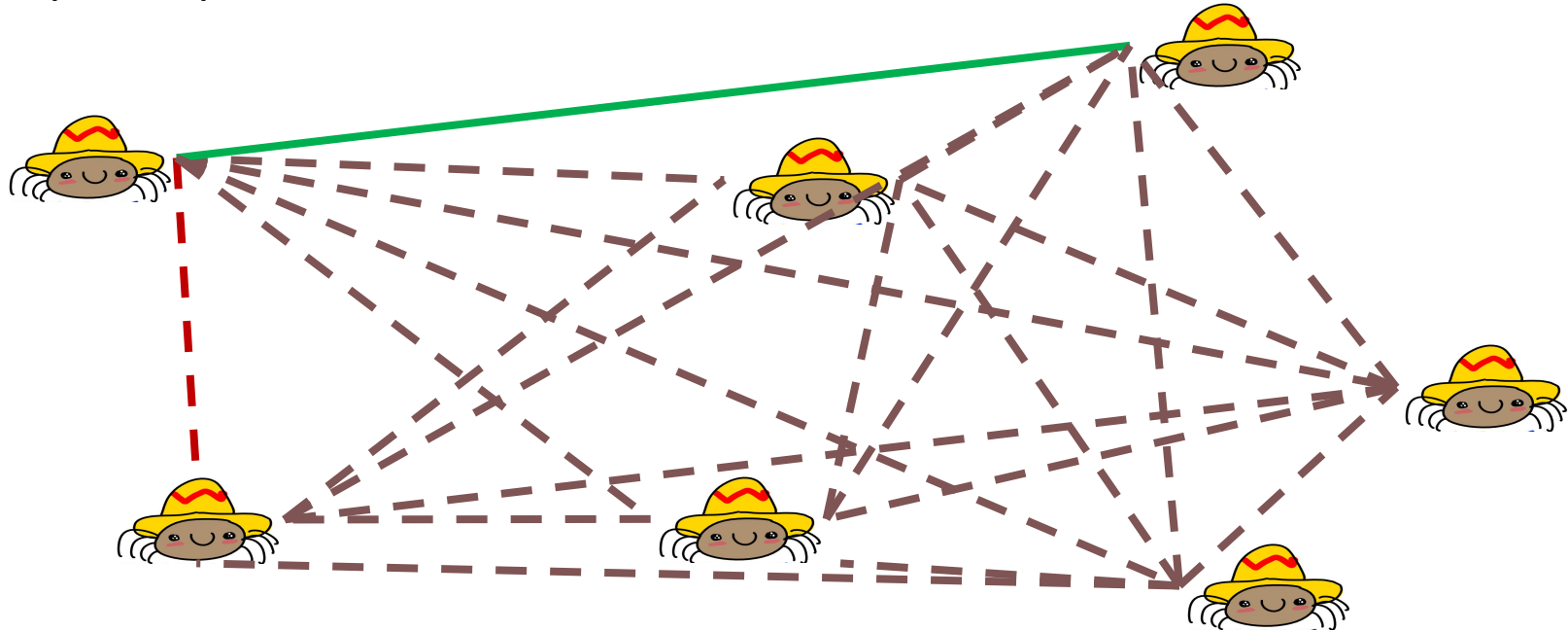
# Gerando uma rede aleatória

$G(7,0.3)$



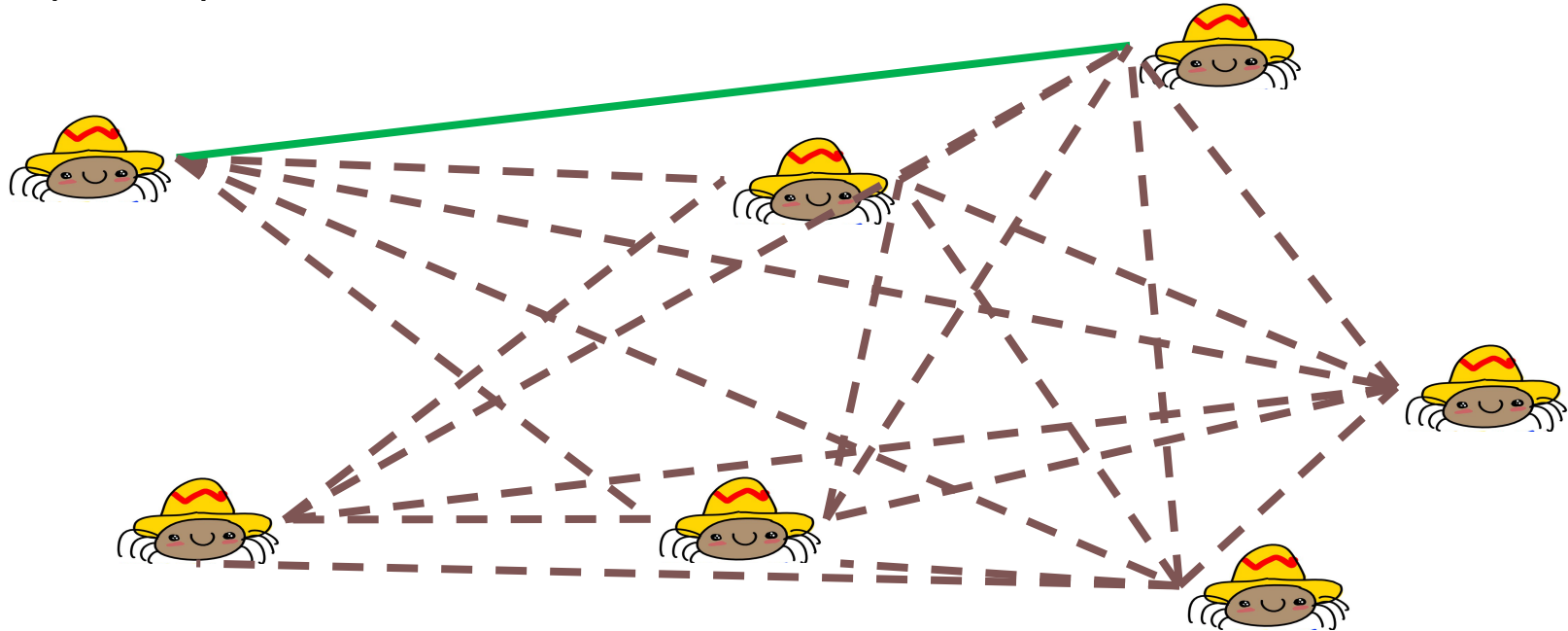
# Gerando uma rede aleatória

$G(7,0.3)$



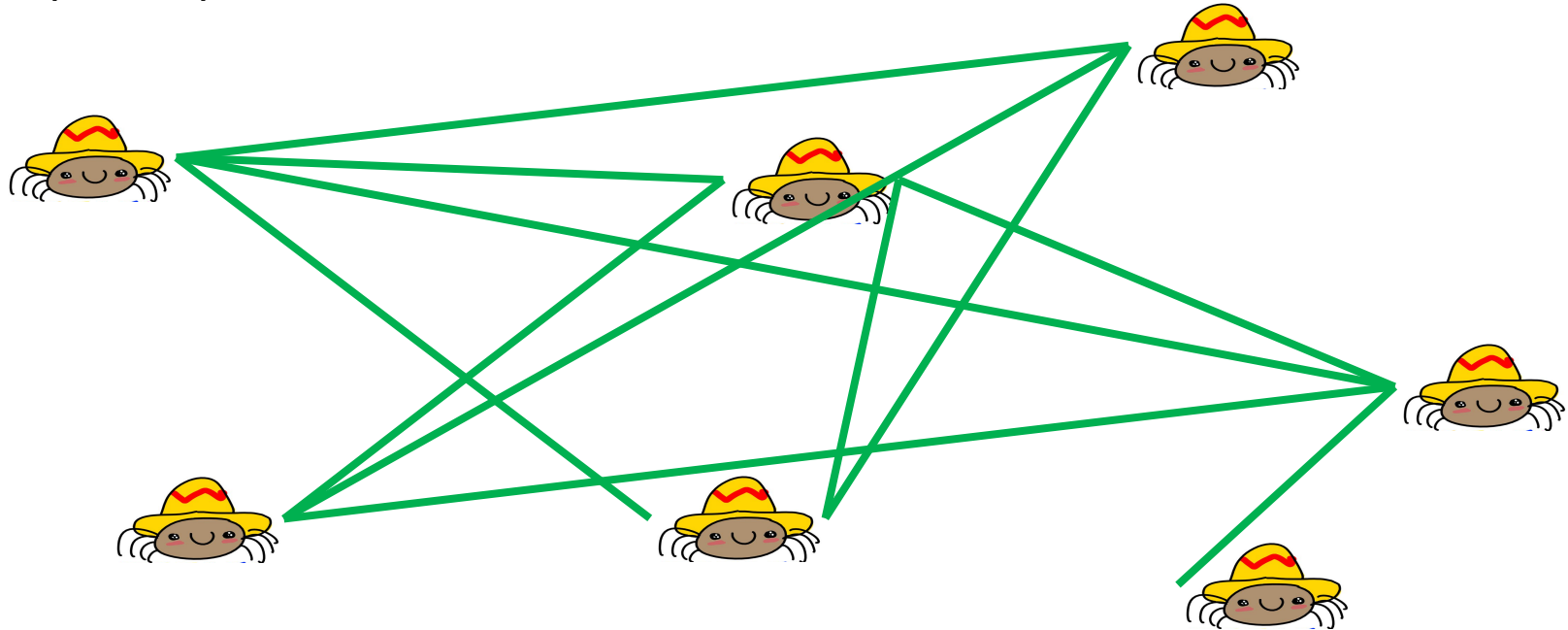
# Gerando uma rede aleatória

$G(7,0.3)$

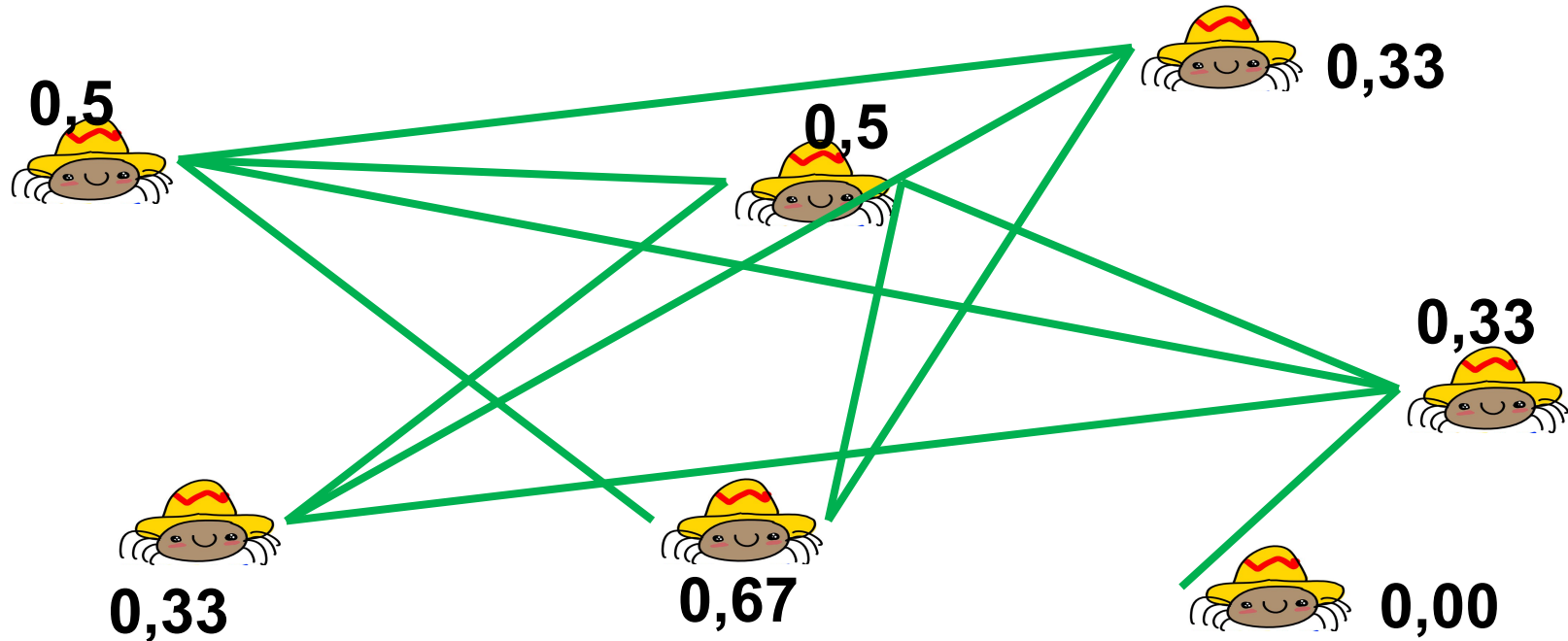


# Gerando uma rede aleatória

$G(7,0.3)$

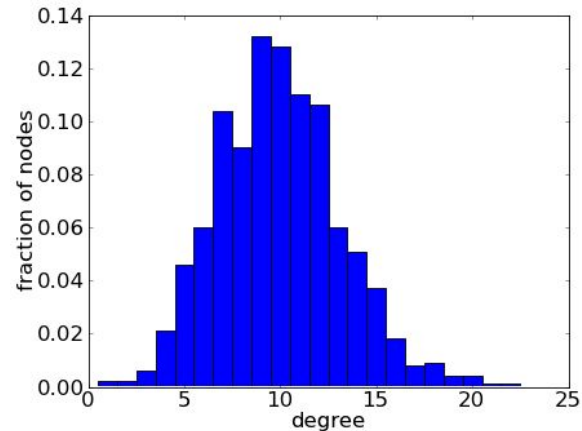


# Gerando uma rede aleatória



# Temos um modelo...?

- ❑ Baixa ocorrência de fechamentos triádicos;
- ❑ Distribuição dos graus .





# Temos um modelo...?

No modelo  $G(n,p)$  esperamos obter  $p \cdot n \cdot (n-1)/2$  arestas ou

$$E(\text{arestas}) = \binom{n}{2} p$$



# Temos um modelo...?

No modelo  $G(n,p)$  esperamos obter  $p \cdot n \cdot (n-1)/2$  arestas ou

$$E(\text{arestas}) = \binom{n}{2} p$$

$P(\text{grau}(v)=k) = ?$



# Temos um modelo...?

No modelo  $G(n,p)$  esperamos obter  $p \cdot n \cdot (n-1)/2$  arestas ou

$$E(\text{arestas}) = \binom{n}{2} p$$

$$P(\text{grau}(v) = k) = \binom{n-1}{k} p^k (1-p)^{n-1-k}$$



# Temos um modelo...?

No modelo  $G(n,p)$  esperamos obter  $p \cdot n \cdot (n-1)/2$  arestas ou

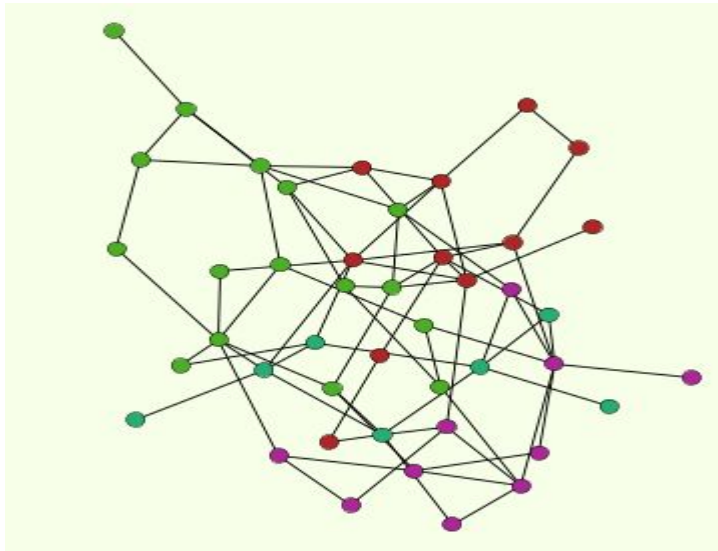
$$E(\text{arestas}) = \binom{n}{2} p$$

$$P(\text{grau}(v) = k) = \binom{n-1}{k} p^k (1-p)^{n-1-k}$$

$$P(\text{grau}(v) = k) \rightarrow \frac{(np)^k e^{-np}}{k!}, n \rightarrow \infty$$



# Temos um modelo...?



Ordem = 50

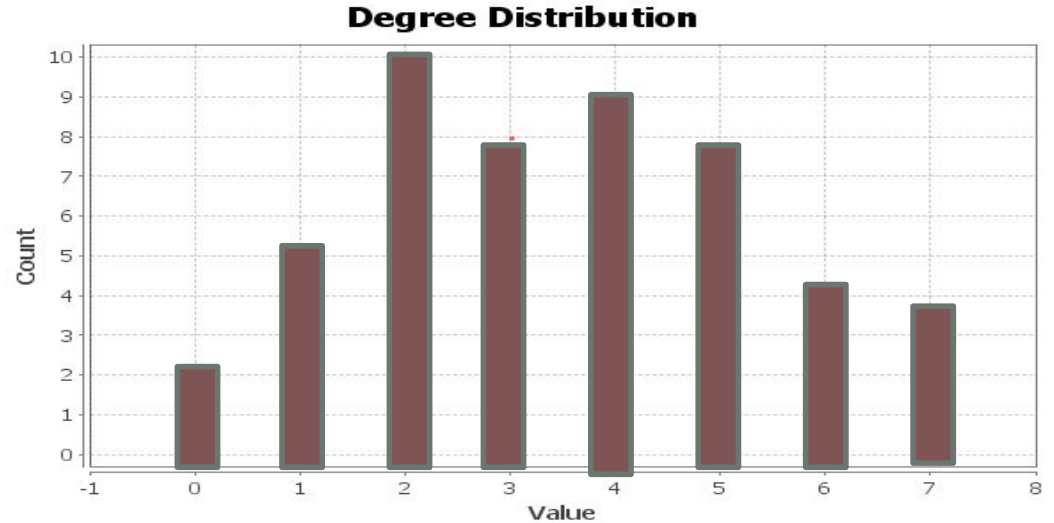
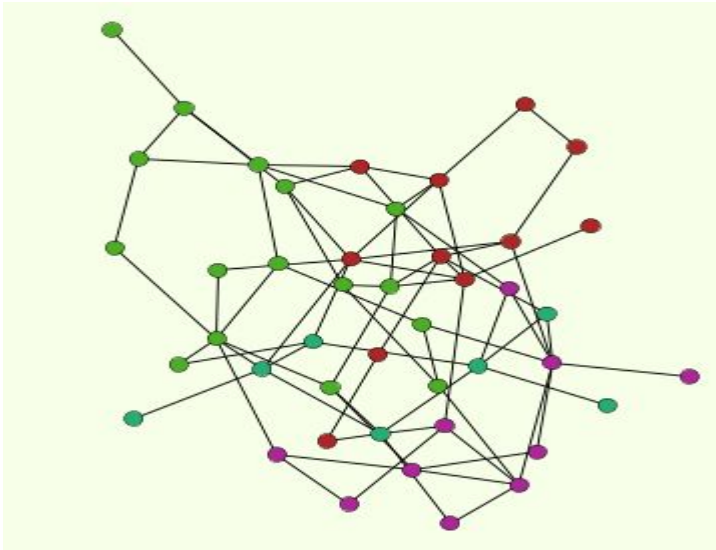
Tamanho = 81

Coef. de Agrupamento Médio = 0,035

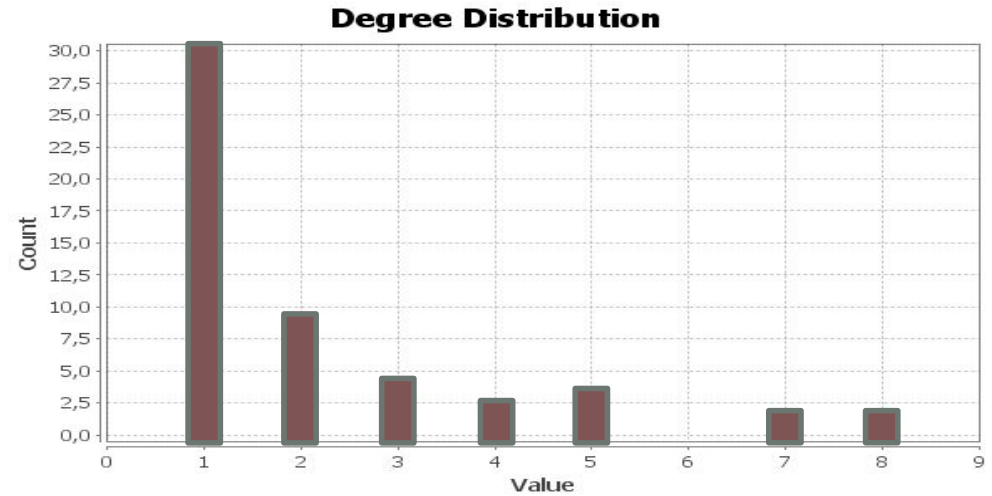
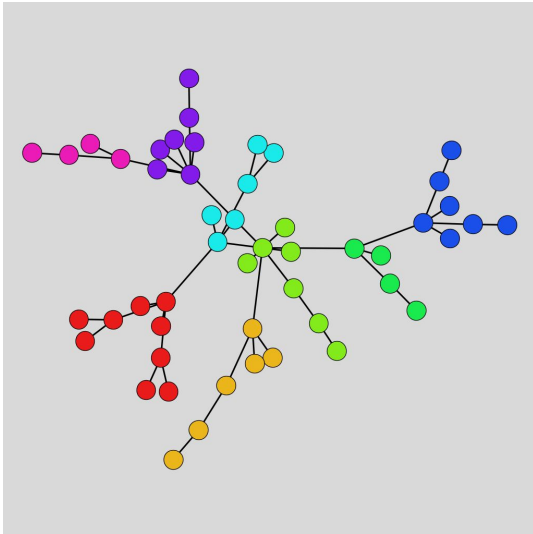
No. de Fechamentos Triádicos = 12



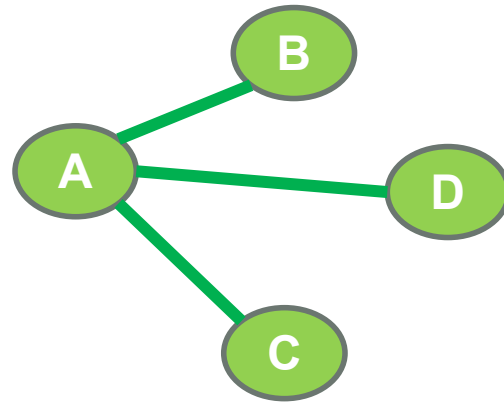
# Temos um modelo...?



# Temos um modelo...?

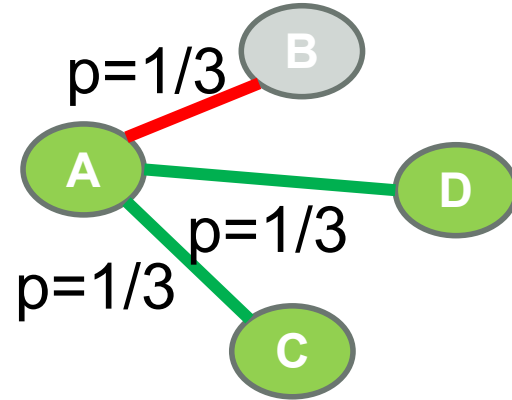


# Temos um modelo...?



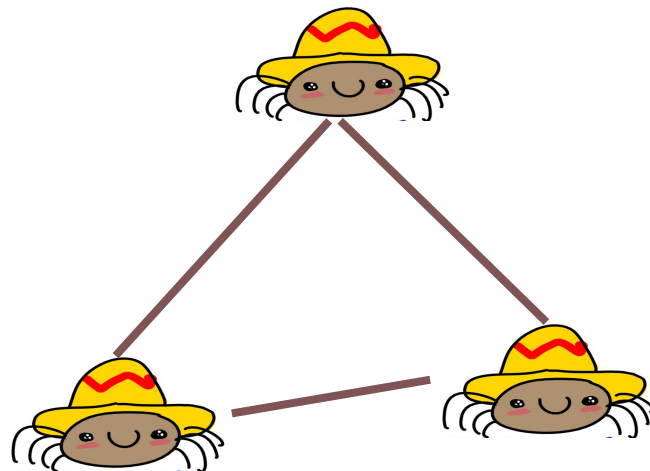


# Temos um modelo...?





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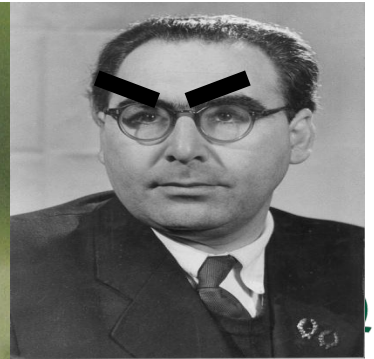


# REDES MUNDO PEQUENO

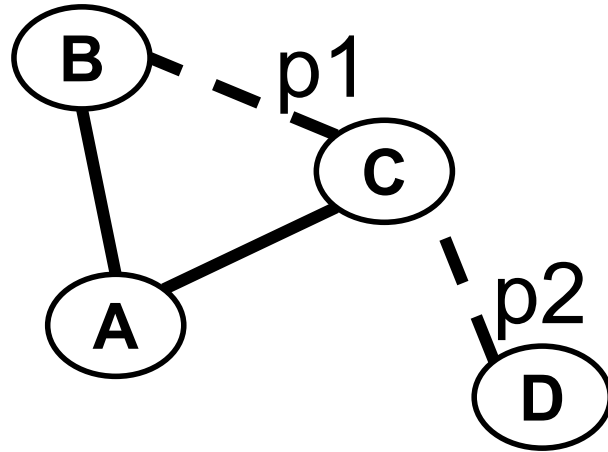
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# Redes Mundo Pequeno



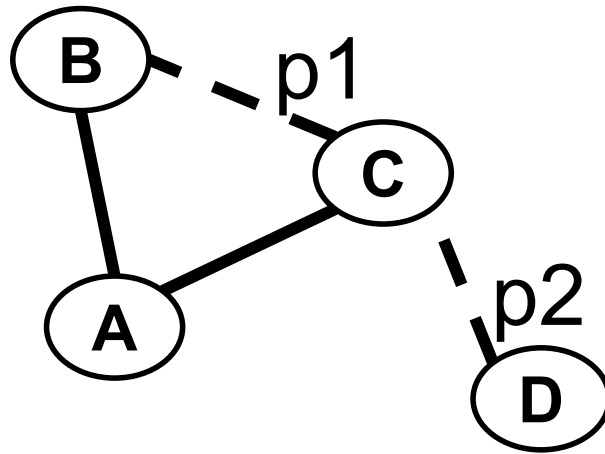
# Redes Mundo Pequeno



$$p1 = p2$$



# Redes Mundo Pequeno



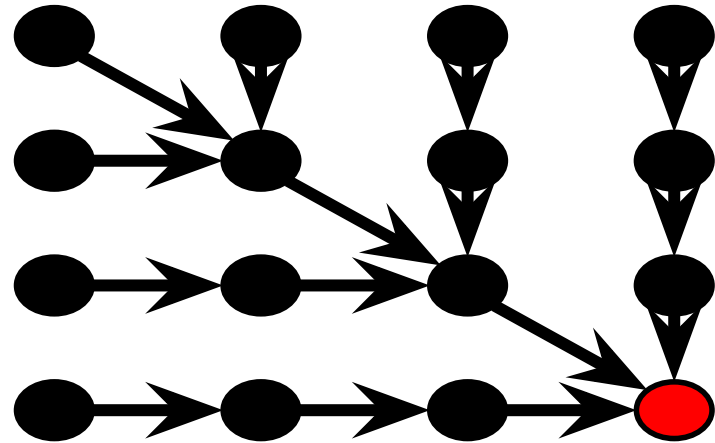
$p1 > p2$



# Cadeias (Chains)

Cidades otimizadas:

- ❑ Trânsito
- ❑ Vizinhança
- ❑ Distribuição demográfica



# Cadeias (Chains)

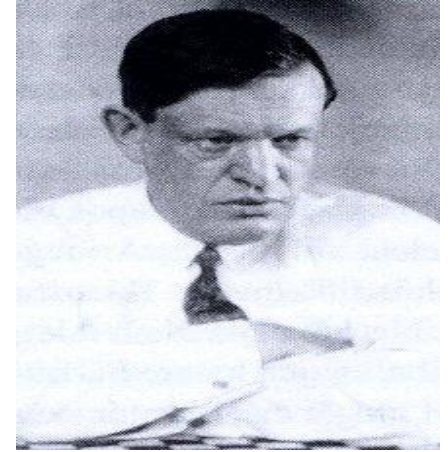
KARINTHY  
FRIGYYES

---

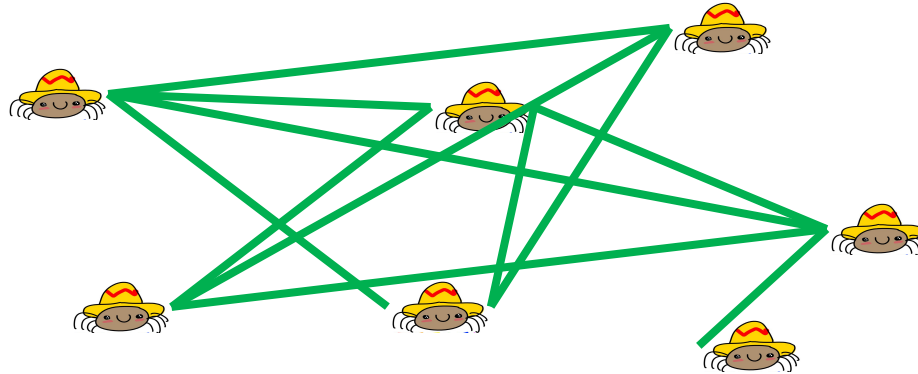
*Chains*

---

SZÉPIRODALMI  KÖNYVKIADÓ



# Cadeias (Chains)





# Cadeias (Chains)

“Vamos selecionar qualquer pessoa entre o 1,5 bilhão de habitantes da Terra, qualquer um em qualquer lugar.

Aposto que usando não mais do que 5 pessoas, uma delas sendo um conhecido dele, eu conseguirei contactar a pessoa escolhida.”.



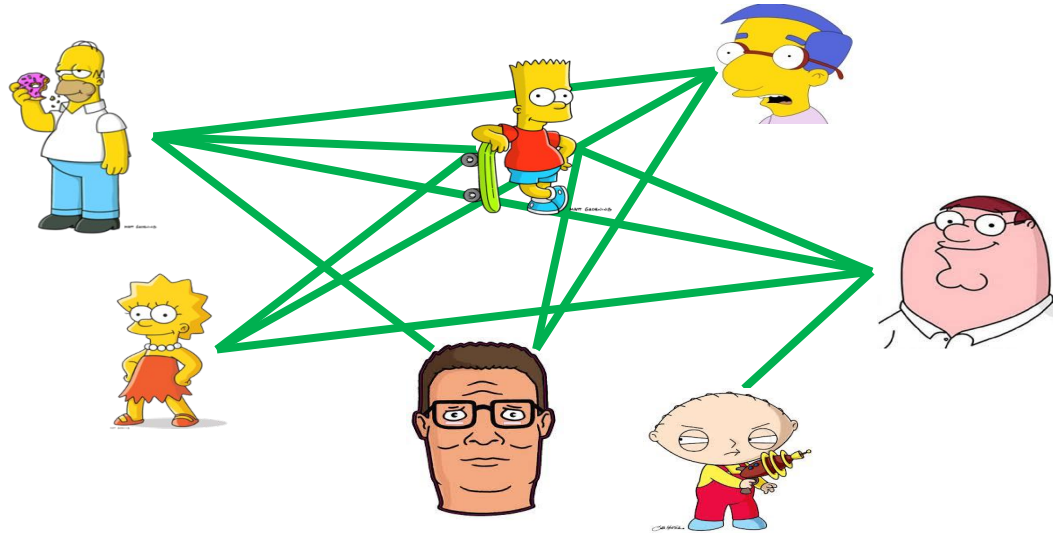
# Experimentos de Mundo Pequeno

Pool & Kochen em 1958

- ❑ Probabilidade de duas pessoas aleatórias possuírem um amigo em comum
- ❑ Distância média entre as pessoas



# Experimentos de Mundo Pequeno



# Experimento de Milgram



# Experimento de Milgram

Se você não conhece pessoalmente a pessoa destino, não tente contatá-la diretamente. Envie essa carta para um conhecido pessoal que **tem mais chances do que você** de conhecer a pessoa destino.



Milgram, *Psych Today* **2**, 60 (1967)



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# Experimento de Milgram

**42/160** cartas.

Menor caminho = 1

Maior caminho = 10

Média caminho = 5,5

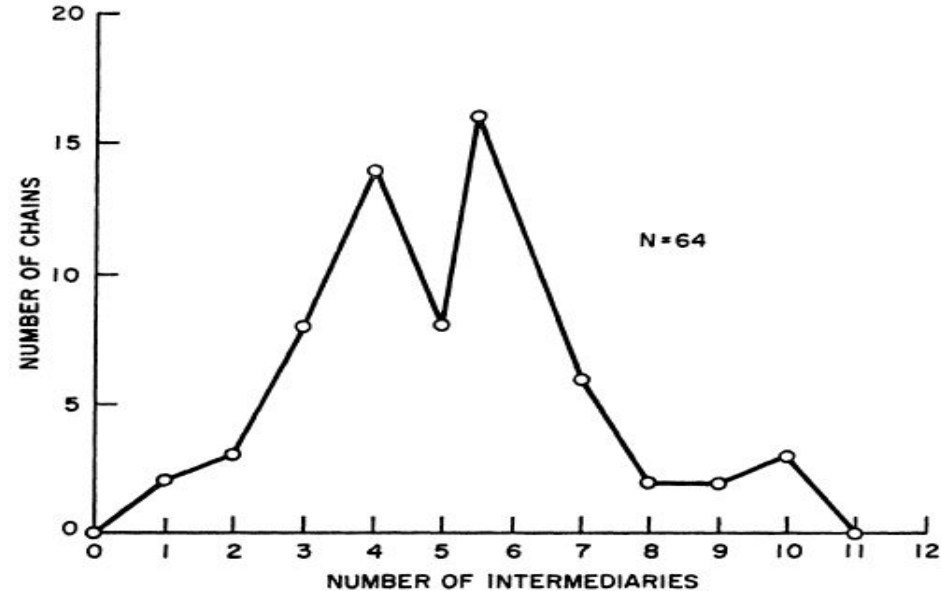


FIGURE 1

*Lengths of Completed Chains*



# Experimento de Milgram

**SEIS GRAUS DE SEPARAÇÃO, 1990**



John Guare



# Mundo Pequeno

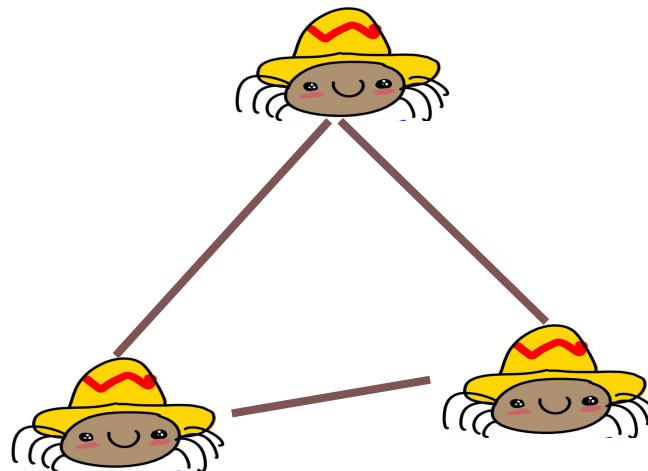
O termo **MUNDO PEQUENO** diz respeito ao fato de que muitas redes reais, embora tenham um número grande de nós, existe um **CAMINHO DE TAMANHO PEQUENO** entre quaisquer dois nós.







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# SEIS GRAUS DE ERDÖS

---

Prof. Fabrício Olivetti de França

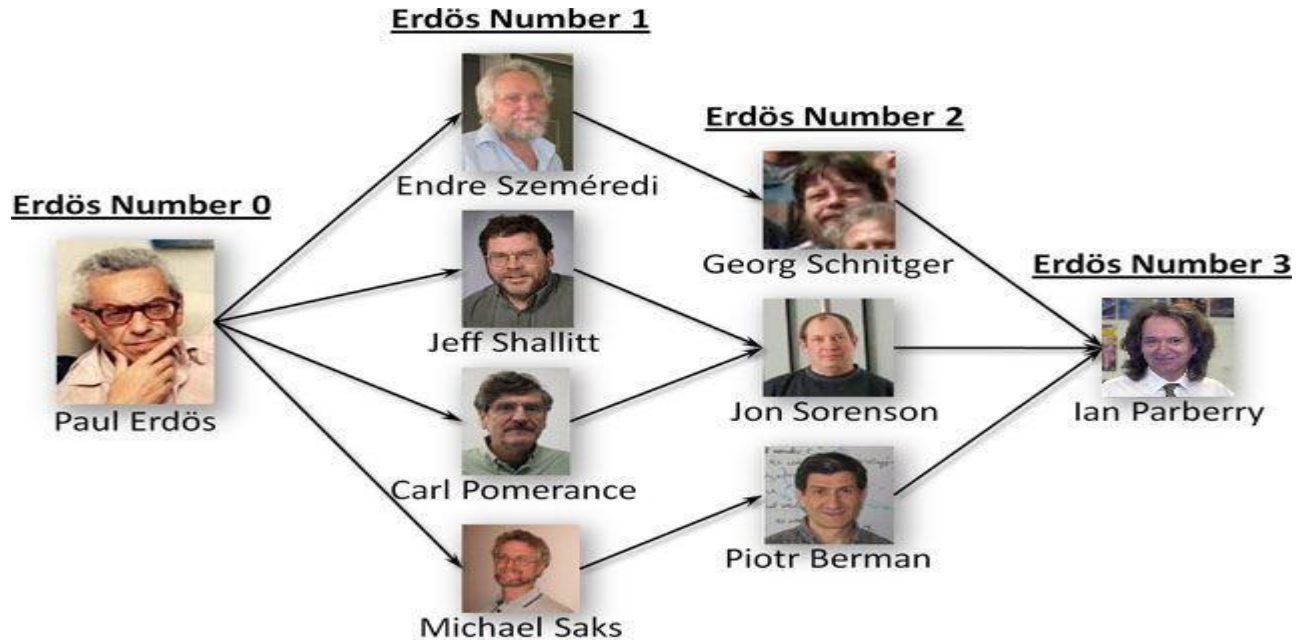
# Os Seis Graus de Erdős

Número de Erdős:

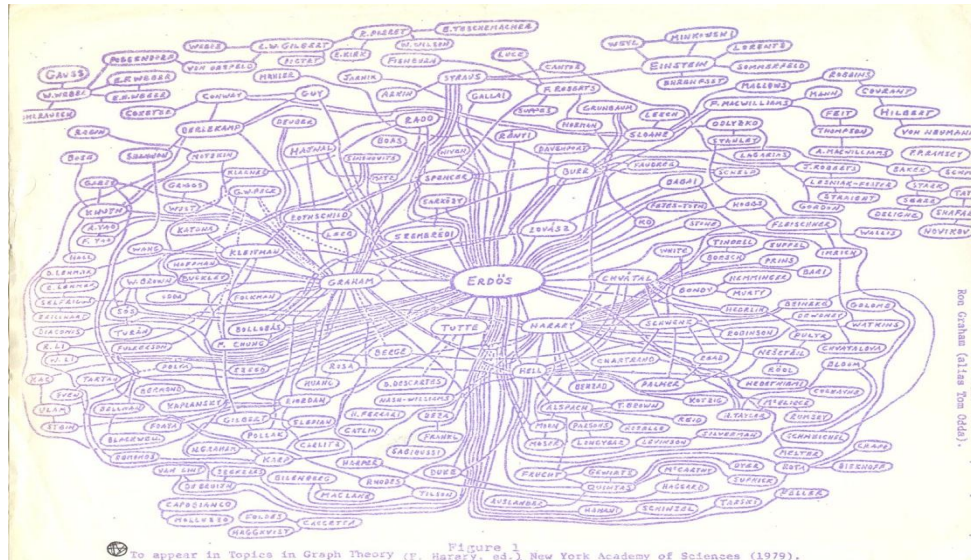
- ❑ Quem escreveu um artigo com Erdős = 1.
- ❑ Quem escreveu com alguém com número um = 2
- ❑ ...



# Os Seis Graus de Erdős



# Os Seis Graus de Erdős



<http://www.oakland.edu>



# Os Seis Graus de Erdős

<http://www.ams.org/mathscinet/collaborationDistance.html>

## MR Erdos Number = 5

<a href="#">Fabrício Olivetti de França</a>	coauthored with	<a href="#">Fernando José Von Zuben</a>	MR2506290
<a href="#">Fernando José Von Zuben</a>	coauthored with	<a href="#">Antônio Carlos Moretti</a>	MR3118526
<a href="#">Antônio Carlos Moretti</a>	coauthored with	<a href="#">Earl R. Barnes</a>	MR1673081
<a href="#">Earl R. Barnes</a>	coauthored with	<a href="#">Alan J. Hoffman</a>	MR1158814
<a href="#">Alan J. Hoffman</a>	coauthored with	<a href="#">Paul Erdős<sup>1</sup></a>	MR0565328

[Change First Author](#)

[Change Second Author](#)

[New Search](#)



# Os Seis Graus de Bacon

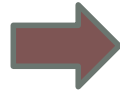


<http://oracleofbacon.org/>



# Os Seis Graus de Bacon

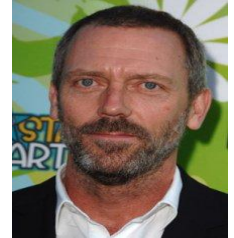
**The Great New Wonderful**



**Mystic River**



**Arthur Christmas**



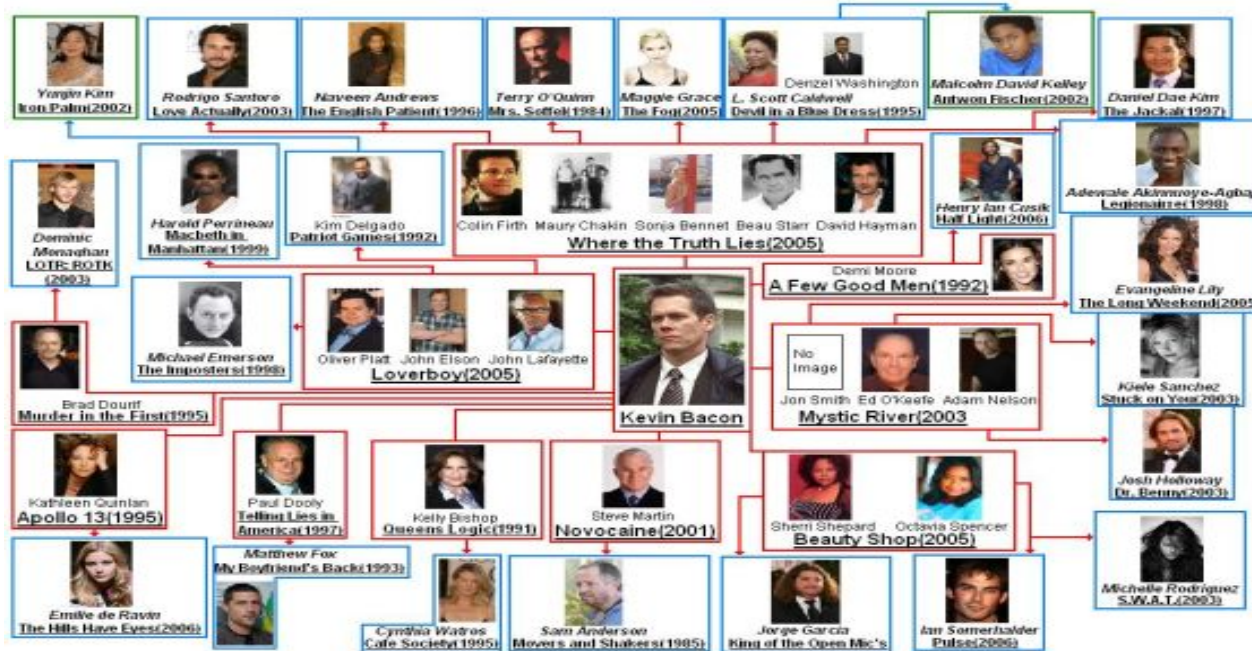
**Sleepers**



**Inglourious Basterds**

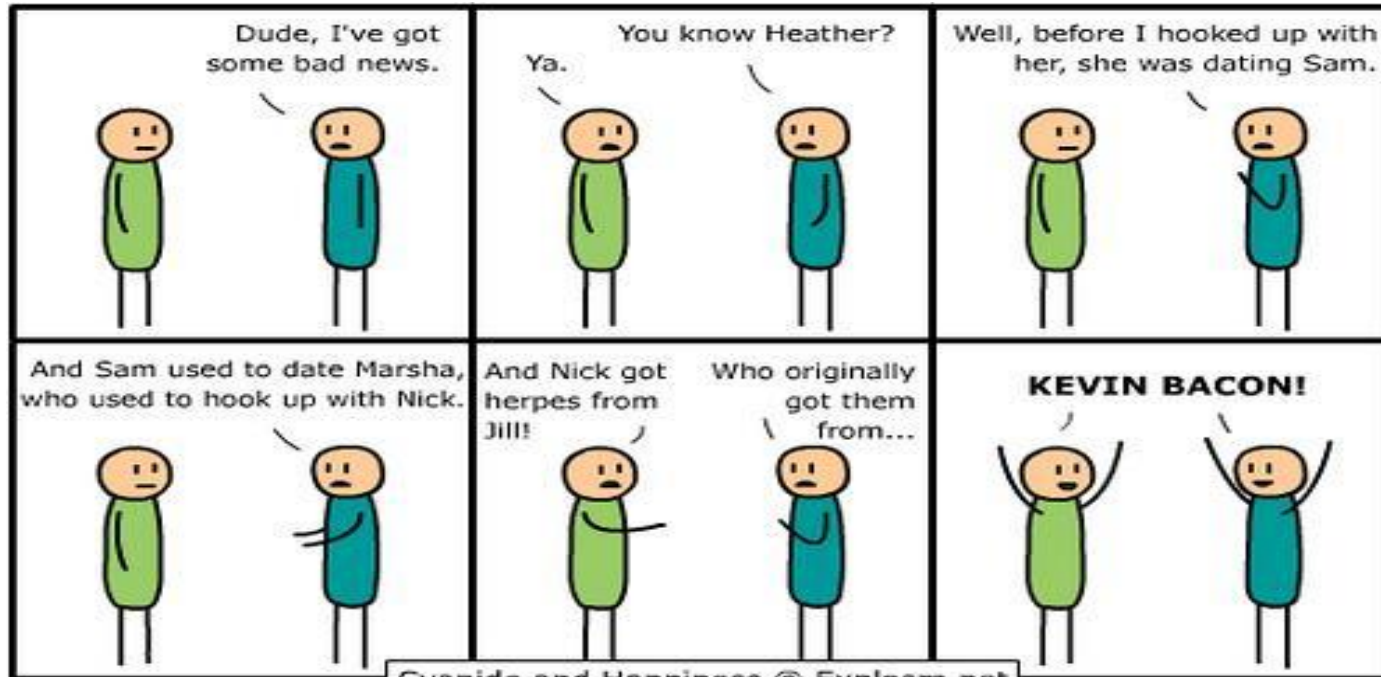


# Os Seis Graus de Bacon





# Os Seis Graus de Bacon



# Os Seis Graus de Bacon

Existem pessoas com número de Bacon **E** número de Erdős!!!



Bruce Reznick

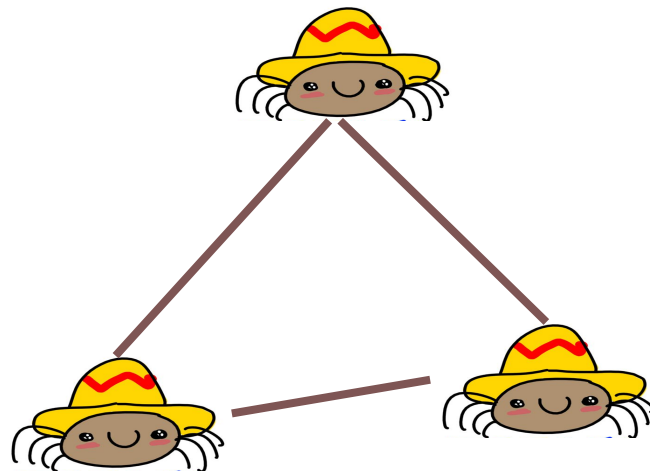
Mais informações:

[http://en.wikipedia.org/wiki/Erd%C5%91s%E2%80%93Bacon\\_number](http://en.wikipedia.org/wiki/Erd%C5%91s%E2%80%93Bacon_number)





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# MODELO MUNDO PEQUENO

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Prof. Fabrício Olivetti de França

# Rede mundo pequeno



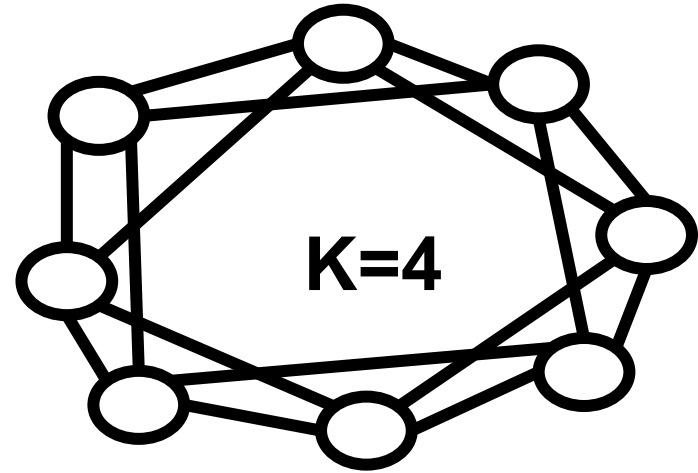
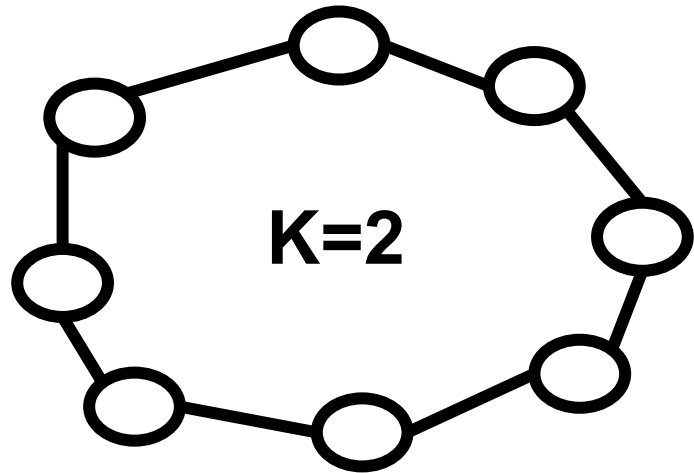
Watts



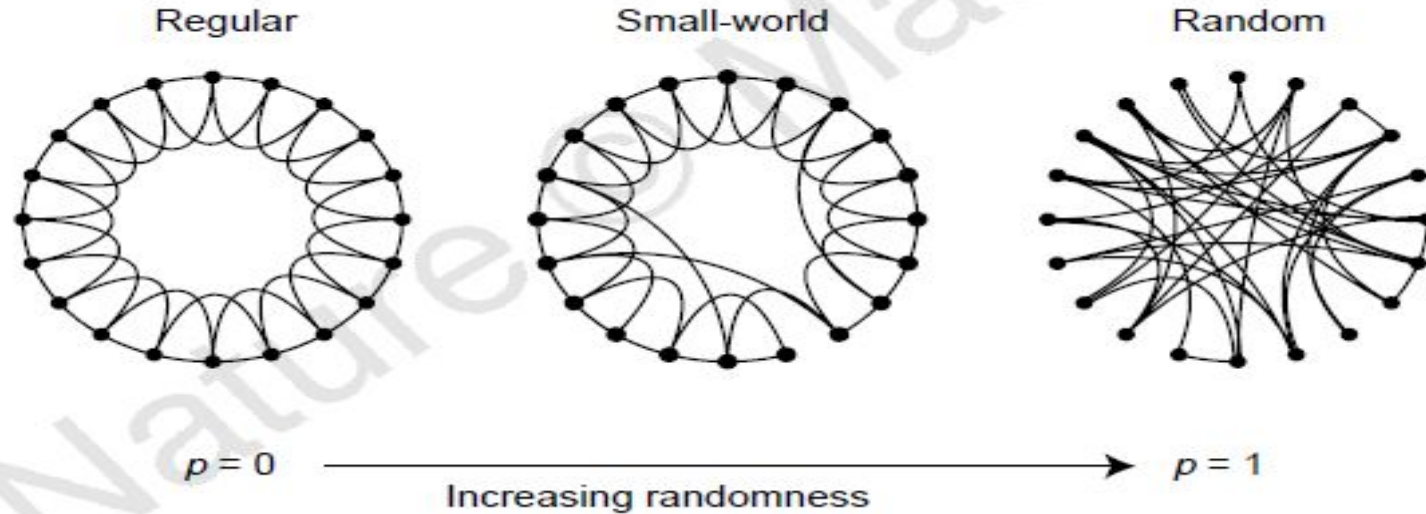
Strogatz



# Rede mundo pequeno



# Rede mundo pequeno

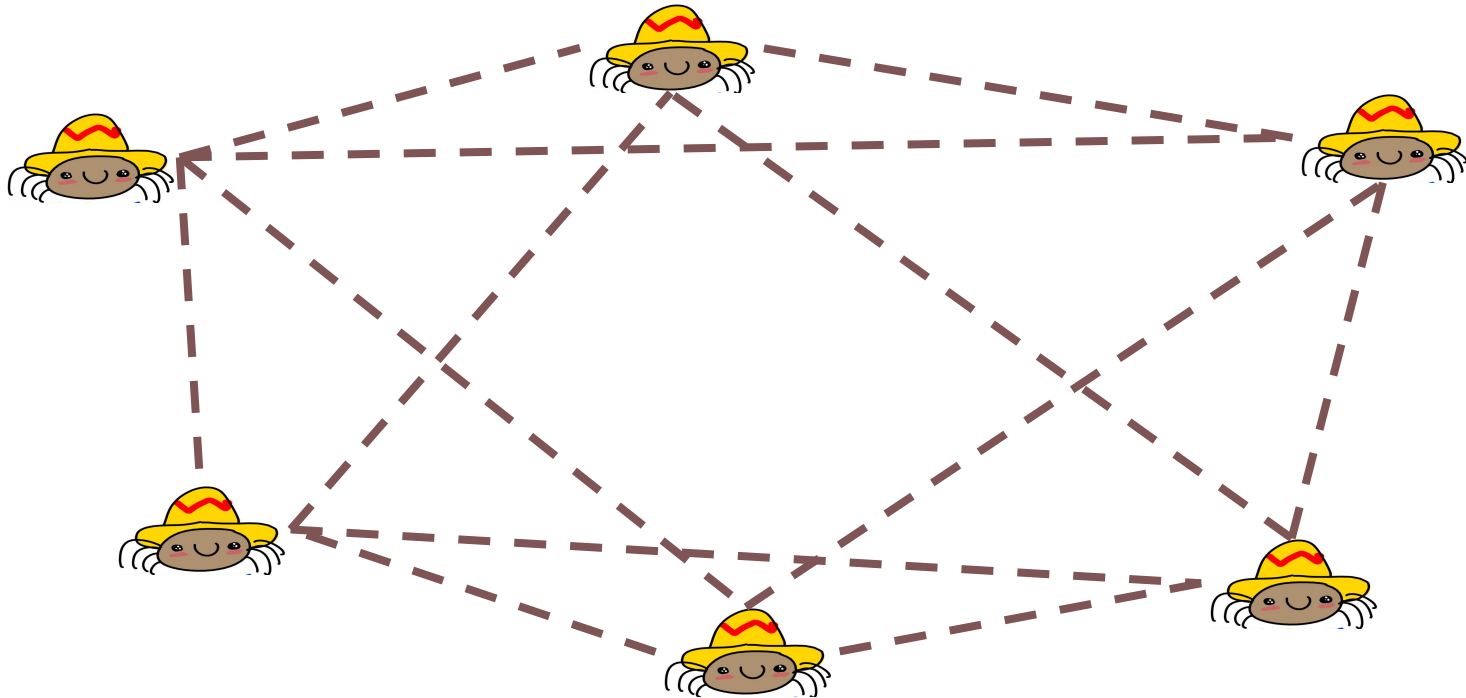


D. J. Watts and S. H. Strogatz, Collective dynamics of “smallworld” networks, *Nature*, 393 (1998), pp. 440–442.

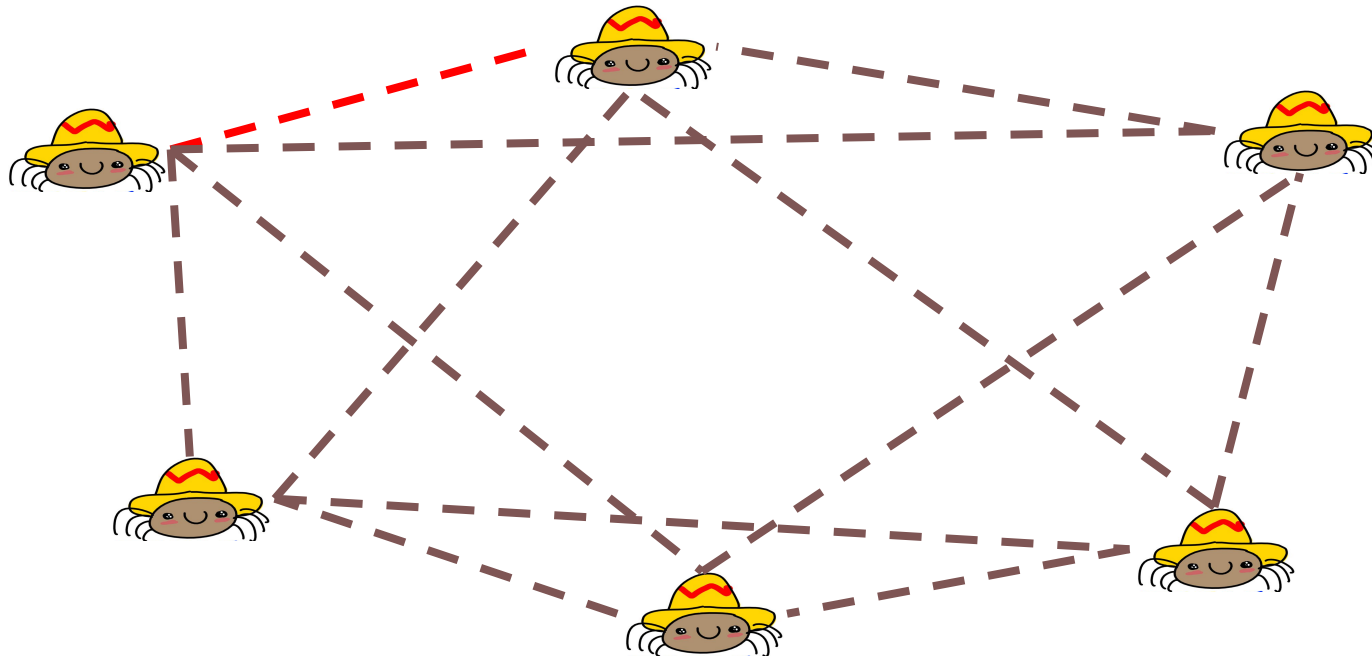


# Rede mundo pequeno

**G(6, 4, 0.3)**: 6 nós, grau inicial = 4, probabilidade = 30%

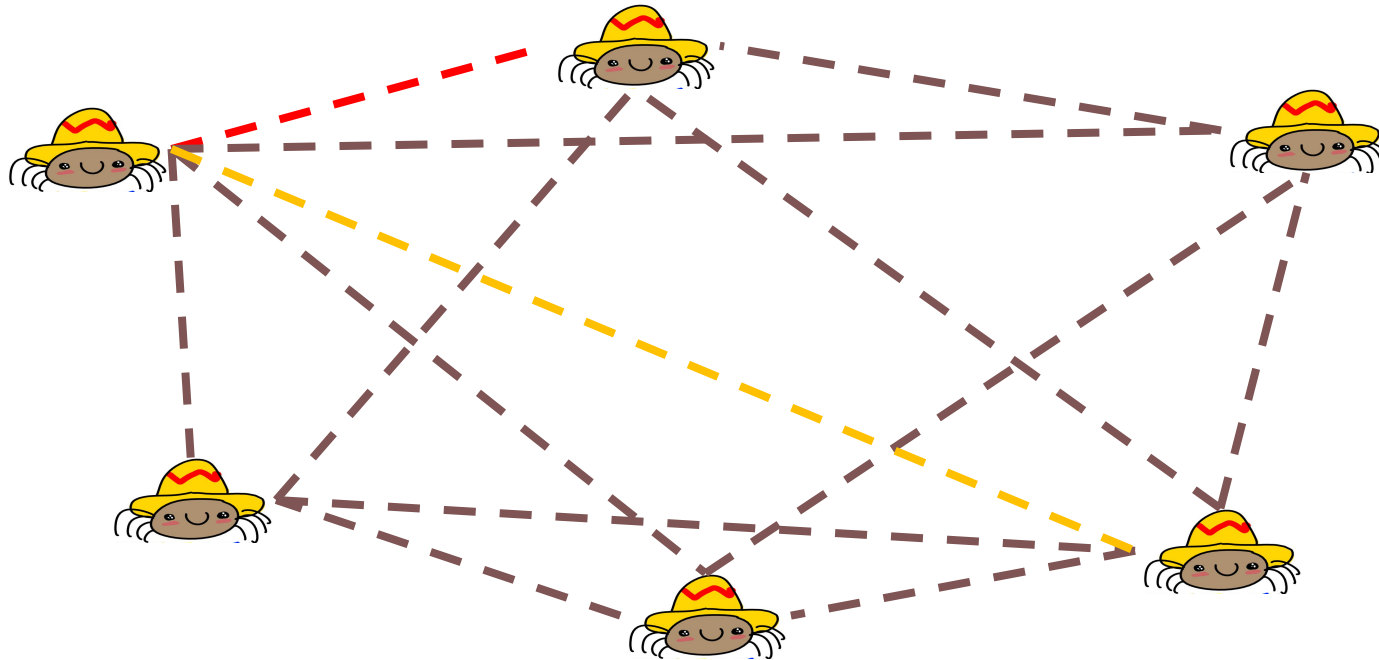


# Rede mundo pequeno

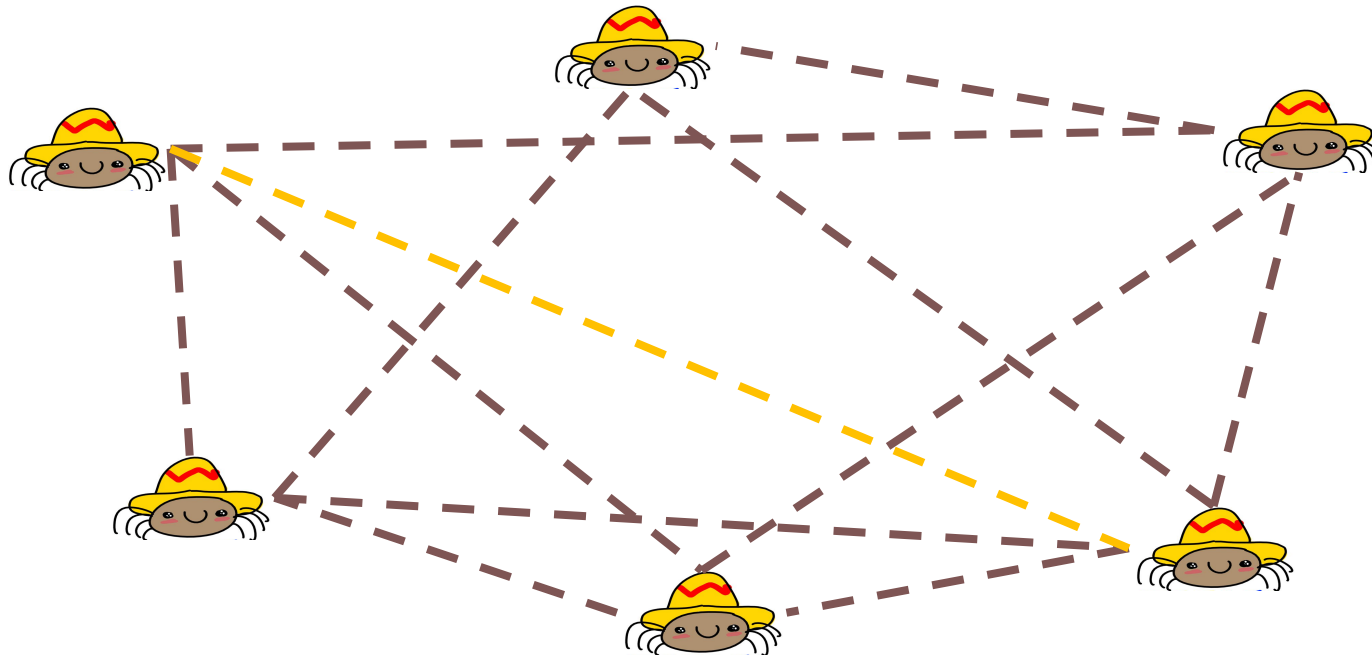




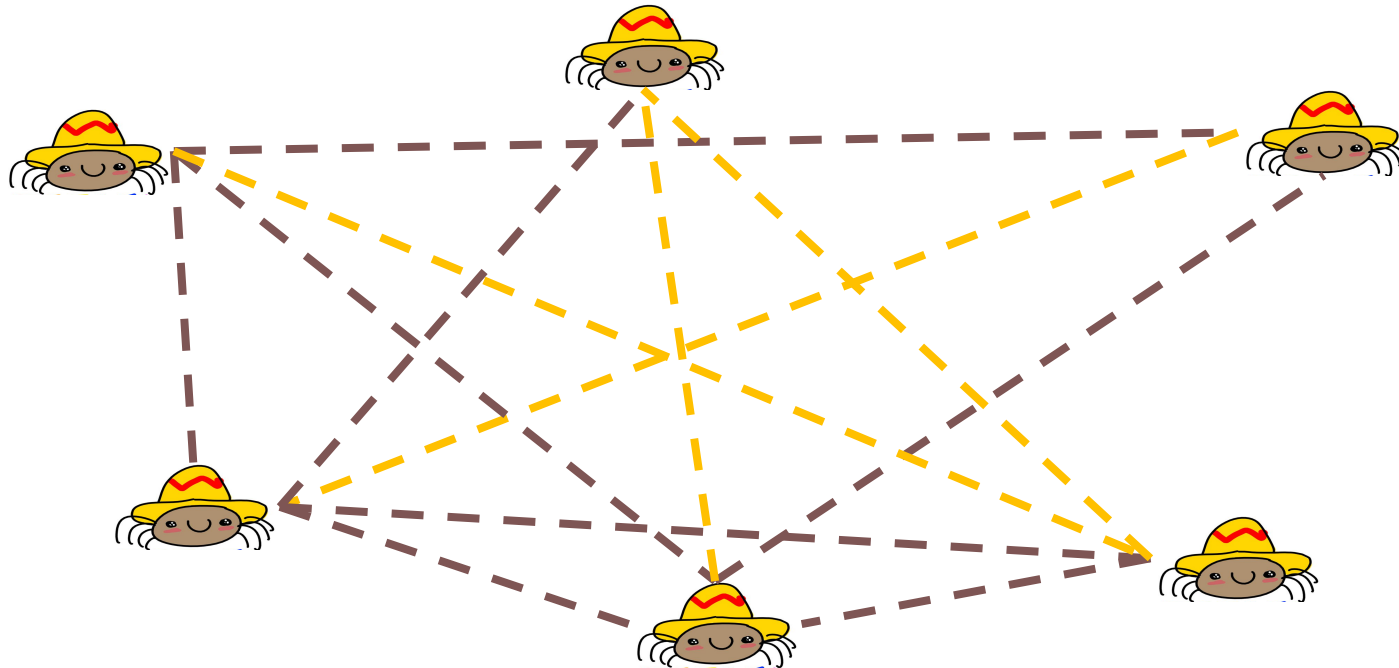
# Rede mundo pequeno



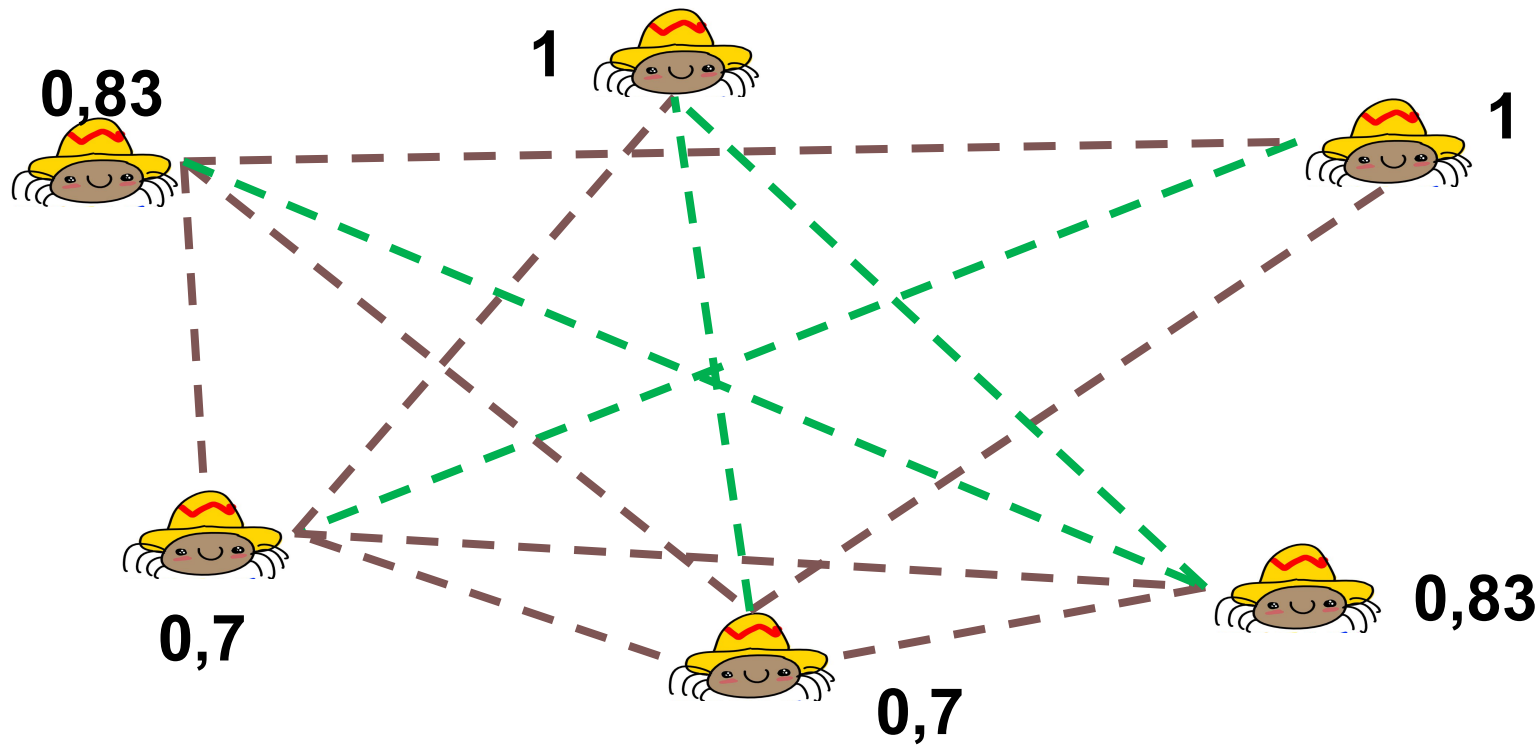
# Rede mundo pequeno



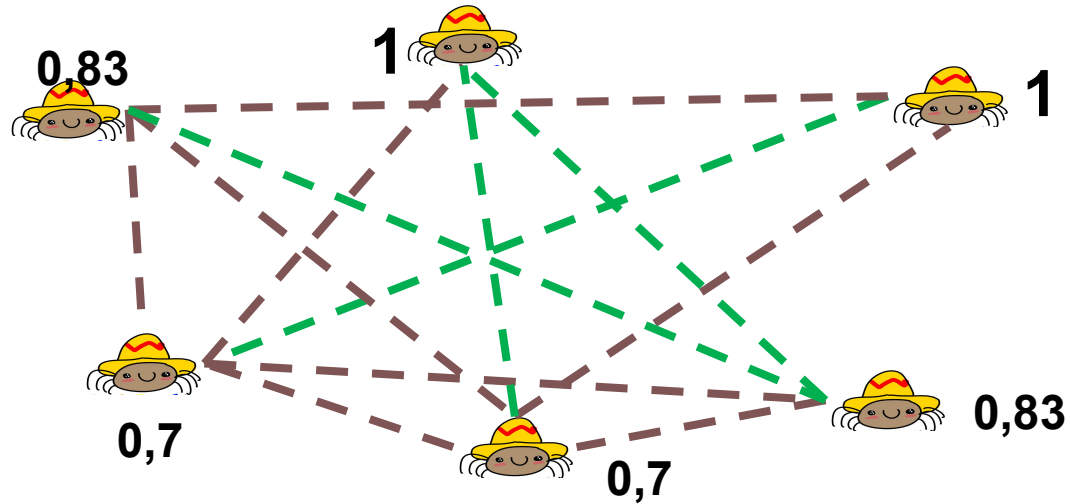
# Rede mundo pequeno



# Rede mundo pequeno

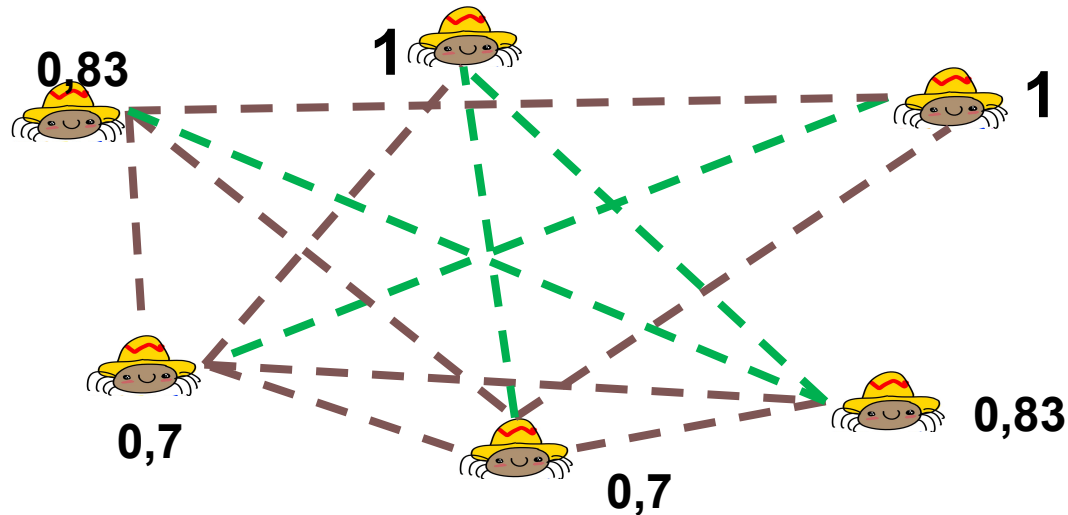


# Rede mundo pequeno



# Rede mundo pequeno

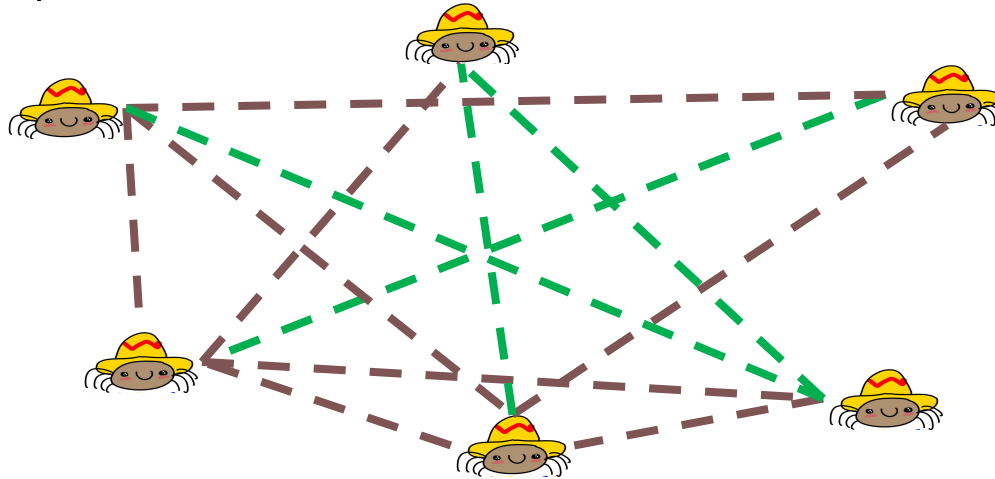
$$p = \frac{2E}{n(n-1)}$$



# Rede mundo pequeno

$$p = 12 / (6 * 5 / 2) = 12 / 15 = 0,8$$

$$C = 0,84$$



# Rede Mundo Pequeno

- ❑ Distância média entre  $n/2k$  e  $\ln(n)/\ln(k)$  para  $p$  entre 0 e 1
- ❑ Coeficiente de agrupamento  $\approx 3(1-p)^3/4$
- ❑ Distribuição do grau de poisson.

